References

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Paul's Notes: → http://tutorial.math.lamar.edu/
DE_complete.pdf
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MAS182 Textbook → Calculus with Applications by Lial Greenwell Ritchey
MAS161 Text book → Calculus 6E by James Stewart
Linear algebra for Calculus – by K.Heuvers
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MAS208 Textbook → Differential Equations – by Dennis G. Zill
ENG267 Textbook → Process Dynamics Modeling and Control by Babatunde A. Ogunnake and W. Harman Ray
Calculator → CASIO fx – 9860G AU

Cost Revenue and Profit

C(x) = Total Manufacturing cost
R(x) = Revenue
P(x) = Profit
X = No. of units
F = Fixed Cost
C = Production cost per unit
S = Sell cost per unit

So

C(x) = cx + F
R(x) = sx
P(x) = R(x) – C(x)

Point Slope form $m=\frac{\Delta y}{\Delta x}=\frac{y_2-y_1}{x_2-x_1}$

Y slope intercept form – (b = the y intercept where x=0) $y=mx+b$
b = y intercept

Point – slope form $y-y_1=m\left(x-x_1\right)$ - p15
Where y_1 and x_1 are points on the line to plug in.

Distance $d=\sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$

Line of Symmetry\ $x=\frac{-b}{2a}$

Perpendicular lines $m_1=-\frac{1}{m^2}$ Rearrange Formula

$m_1 \cdot m_2=-1$

Domain is a set of x values (**independent** Variable) and
Range is a set of possible y values (**dependant** variable)

Inverse of a 2*2 matrix – Larger Matricis done by calculator

Identity = $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ The purpose of an inverse matrix is to cancel itself out when solving an equation of matrices

Calculator : OPTN → F2 MAT → F6 → F1 Iden → size of matrix → F6 → F1 MAT → ALPHA → choose letter.

Eg : The inverse of A is A^{-1}

$$A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix} \quad A^{-1}=\begin{bmatrix}-2 & 1\\ \frac{3}{2} & -\frac{1}{2}\end{bmatrix}$$

$$AA^{-1}=I \quad \begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}\cdot\begin{bmatrix}-2 & 1\\ \frac{3}{2} & -\frac{1}{2}\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

Say $A=\begin{bmatrix}1 & 1\\ 2 & 3\end{bmatrix} \quad x=\begin{bmatrix}x\\ y\end{bmatrix} \quad B=\begin{bmatrix}4\\ 11\end{bmatrix}$

$$AX=B$$
$$\Rightarrow A^{-1}AX=A^{-1}B$$
$$\Rightarrow X=A^{-1}B$$
$$\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}3 & -1\\ -2 & 1\end{bmatrix}\begin{bmatrix}4\\ 11\end{bmatrix}$$
$$\Rightarrow \begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1\\ 3\end{bmatrix}$$

To find the inverse change $\begin{bmatrix}a & b\\ c & d\end{bmatrix}$ to $\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$

Then $A^{-1}=\frac{1}{ad-bc}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$

To find the inverse on a calculator

OPTN Mat **F2** Mat **F1 SHIFT** x^{-1} **)**

Matrix written as (Row, Column)
Eg. Size of a Multiplication of matrices (3 X 2) and (2 x 4)
The columns in A must equal the rows in B
So the above can multiply and the result is a (3 X 4) matrix.
For $A^{-1}=0$ the ad – bc section above must = 0

Matrix example

$$\begin{bmatrix}3 & 0 & 1\\ 1 & -2 & 0\end{bmatrix}\begin{bmatrix}-1 & 2\\ 0 & -1\\ 4 & 2\end{bmatrix}=\begin{bmatrix}1 & 8\\ -1 & 4\end{bmatrix}$$

Multiply top row of A by first column of B. Then by second column of B. Then bottom row of A by first Column of B. Then second column of B.

Det(A) = |A|

Det(A) = 0 is a singular Matrix
$Det(A)\neq 0$ is a non singular Matrix and inverse exists.
On calculator : OPTN → MAT F2 → DET F3

Determinant of order 2 - Pg 822 stew

$$\begin{vmatrix}a & b\\ c & d\end{vmatrix}=ad-bc$$

Determinant of order 3 - Pg 822 stew

$$\begin{vmatrix}a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\\ c_1 & c_2 & c_3\end{vmatrix}=a_1\begin{vmatrix}b_2 & b_3\\ c_2 & c_3\end{vmatrix}-a_2\begin{vmatrix}b_1 & b_3\\ c_1 & c_3\end{vmatrix}+a_3\begin{vmatrix}b_1 & b_2\\ c_1 & c_2\end{vmatrix}$$

Determining whether a matrix has an inverse Heuv pg 67
If det A != 0 then the inverse A^{-1} exists

Matrix Transpose pg APP7 Zill
Where the rows become the columns and the columns become the rows.

Eigen Values Pauls notes LinAlg_Complete.pdf pg 305
Ha's hand written notes
systems_of_linear_equations_ha_hand_written_notes.pdf

- We then call **x** an eigenvector of A and **λ** an eigenvalue of A.
  $(\lambda I-A)x=0$ pg 315 pauls.
- Will need long division of polynomials – Pauls notes Alg_Complete.pdf pg245
-Definition 4 The set of all solutions to (λ I − A) x = 0 is called the eigenspace of A corresponding to λ .

Example:

$B=\begin{bmatrix}3 & 1\\ 4 & 0\end{bmatrix}$ det (b – λI)

$$det\begin{bmatrix}3-\lambda & 1\\ 4 & 0-\lambda\end{bmatrix}$$
= (3 - λ)(-λ) – 4
= λ^2 - 3λ – 4
= (λ + 4) (λ + 1)

Eigenvalues **= λ = 4, λ = -1**

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Then to get EigenVectors for each Eigenvalue :-
Insert **λ = 4** into equation above

$$\begin{bmatrix} 3-4 & 1 \\ 4 & 0-4 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 4 & -4 \end{bmatrix}$$

Solve for (b – λI)=0

$$\begin{bmatrix} -1 & 1 \\ 4 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

Augment the matrix

$$\left(\begin{array}{cc|c} -1 & 1 & 0 \\ 4 & -4 & 0 \end{array}\right) \qquad \textbf{(1.1)}$$

Do row operations on $R_2 \rightarrow R_2 + 4R_1 = 0$
-1x + 1y = 0 → x = y
Set y = t

So EigenVector for **λ = 4** is $t\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Then substitute **λ = -1** into equation above. Skip to **(1.1)** to get the

Augmented = $\left(\begin{array}{cc|c} 4 & 1 & 0 \\ 4 & 1 & 0 \end{array}\right)$

Do row operations on $R_2 \rightarrow R_2 - R_1 = 0$
4x + 1y = 0 → x = -1/4y
Set y = t

So EigenVector for **λ = 4** is $t\begin{bmatrix} -\frac{1}{4} \\ 1 \end{bmatrix}$ or multiple of is $t\begin{bmatrix} -1 \\ 4 \end{bmatrix}$

To find EigenValues of 3x3 matrix

$$A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Characteristic Equation

$$-\lambda^3 + (a+e+i)\lambda^2 - \left(\begin{vmatrix} e & f \\ h & i \end{vmatrix} + \begin{vmatrix} a & c \\ g & i \end{vmatrix} + \begin{vmatrix} a & b \\ d & e \end{vmatrix}\right)\lambda + |A|$$

$$\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \times \frac{S}{R}$$

Simultaneous Equations : (see example page ENG262SG20091-1.5)
2 simultaneous equations can be used in a sort of ratio to each other. For Example :

$$a = b \qquad (1)$$
$$x = y \qquad (2)$$

To solve this we can put into the form :

$\frac{a}{x} = \frac{b}{y}$ and then solved like a normal equation from here:

Quadratic Formula $f(x) = ax^2 + bx + c$

Quadratic solution $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ Can have 0,1 or 2 solutions

If the $b^2 - 4ac$ section is
–ve =no solution
0 = one solution
+ve = two solutions

Quadratic Second Rule : pg 161L – 10
eg:

$$x^2 - 6x + 15$$
$x^2 + 2ab + b^2$ where a=x
b = -3
Will have left over 15 - 9 = 6

Therefore $(x-3)^2 + 6$

BIMDAS – work left to right

Brackets
Indicis
Multiply
Divide
Add
Subtract

Line of symmetry used against the quadratic formula $x = -\frac{b}{2a}$

$$\sin\theta = \frac{O}{H} \Rightarrow H\sin\theta = O \Rightarrow H = \frac{O}{Sin\,\theta}$$
$$\cos\theta = \frac{A}{H} \Rightarrow H\cos\theta = A \Rightarrow H = \frac{A}{\cos\theta}$$
$$\tan\theta = \frac{O}{A} \Rightarrow A\tan\theta = O \Rightarrow A = \frac{O}{\tan\theta}$$

eg 7π

$$126^\circ = \frac{0.7\pi}{\pi}(180^\circ) \qquad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$$

$$x^{-2}y^{-2} = \frac{1}{x^2y^2} \qquad (b \neq 0)$$

$$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$$

$$(a^m)^n = a^{m\times n} \qquad a^m \cdot a^n = a^{m+n} \qquad a^{\frac{1}{2}} \cdot a = a^{\frac{3}{2}}$$

$$a^n \cdot b^n = (ab)^n \qquad \frac{a^m}{a^n} = a^{m-n} \quad (a \neq 0)$$

$$\sqrt[n]{a^n} = \begin{cases} |a|.if.n.even \\ a.if.n.odd \end{cases} \qquad \frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt{\frac{a}{b}} \quad (b \neq 0)$$

$$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab} \qquad a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m = \left(a^{1/n}\right)^m$$

$$\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} \qquad \frac{a}{\frac{m}{n}} = \frac{a}{1} \div \frac{m}{n} = \frac{a}{1} \times \frac{n}{m} = \frac{an}{1m}$$

$$\frac{x^{-3}}{y^{-\frac{1}{2}}} = \frac{y^{\frac{1}{2}}}{x^3} \qquad a^{\frac{-m}{n}} = \frac{1}{a^{\frac{m}{n}}} \quad (a \neq 0)$$

$$a^{-n} = \frac{1}{a^n} \qquad a^n = \frac{1}{a^{-n}}$$

$$a^0 = 1 \qquad \sqrt{ab} = \sqrt{a} \times \sqrt{b} \quad \textbf{eg}$$
$$e^{a+b} = e^a \times e^b \qquad \sqrt{160} = \sqrt{16} \times \sqrt{10}$$

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc} \qquad \frac{a}{b} \pm \frac{c}{d} = \frac{ad+bc}{bd}$$

$$ab^n = a^n \times b^n \qquad \textbf{eg} \;\; 160^n = 16^n \times 10^n$$
$$3\cos X = cos3x \qquad (e^{-1+x})^2 = e^{-2+2x}$$

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$$\frac{\frac{a}{b+c}}{ha}=\frac{a}{\frac{ha}{\frac{b}{ha}+\frac{c}{ha}}}$$

$$\frac{1}{\frac{1}{2}+\frac{1}{3}}=\frac{2\times3\times1}{\frac{3\times1}{3\times2}+\frac{2\times1}{3\times3}}=\frac{2\times3}{\frac{3}{6}+\frac{2}{6}}=\frac{2\times3}{3+2}$$

Absolute Numbers

When in graph mode **OPTN** NUM **F5** Abs **F1**

When in standard 'RUN' mode **OPTN** > **F6** NUM **F4** Abs **F1**

2nd Derivative - p753 Used to define whether a curve is concave up or concave down. If $f''(x)>0$ then it's concave up

if $f''(x)<0$ then it's concave down

Graphing the inverse of a function ie swap x and y

After the graph is drawn **F4** then INV **F4**

Differential Calculus : is finding the rate of Change of one quantity with respect to another.

Integral Calculus : Once we know the differential We can find the relationship Between the two.

Example: - Water flowing out of a pipe which is becoming greater.
X axis we have time, Y axis we have flow rate.
The differential (angle) of the graph will give us how rapidly the flow is increasing.
The integral gives the area under the curve, which will give the total volume of the flow with in a time period.

Basic Rules of Differentiation

Rule 1 : Derivative of a constant $\frac{d}{dx}(c)=0$ (c, a constant)

Rule 2 : The Power Rule

If n is any real number then $\frac{d}{dx}(x^n)=nx^{n-1}$

Rule 3 : Derivative of a constant Multiple of a function

$$\frac{d}{dx}[cf(x)]=c\frac{d}{dx}[f(x)] \quad \text{(c, a constant)}$$

Rue 4 : The sum Rule

$$\frac{d}{dx}[f(x)\pm g(x)]=\frac{d}{dx}[f(x)]\pm\frac{d}{dx}[g(x)]$$

Rule 5 : The product Rule – p 654

$$\frac{d}{dx}[f(x)g(x)]=f(x)g'(x)+g(x)f'(x)$$

Rule 6 : The Quotient Rule - p656

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]=\frac{g(x)f'(x)-f(x)g'(x)}{[g(x)]^2} \qquad (g(x)\neq0)$$

Rule 7 : The Chain Rule – p668

If $h(x)=g[f(x)]$ then

$$h'(x)=\frac{d}{dx}g(f(x))=g'(f(x))\times f'(x)$$

Equivalently, if we write $y=h(x)\times g(u)$, where $u=f(x)$, then

$$\frac{d}{dx}=\frac{dy}{du}\cdot\frac{du}{dx}$$

Rule 8 : The General Power Rule

If the function f is differentiable and $h(x)=[f(x)]^u$ (n, a real number) then

$$h'(x)=\frac{d}{dx}[f(x)]^n=n[f(x)]^{n-1}f'(x)$$

Separable differentiantion.

If you can not separate the x and y to opposite sides of the equals sign then you need to do something like this.

When differentiating 'y' with respect to y. it needs to be multiplied by a to even the equation out relative to the 'x'. See the following

$$\frac{dy}{dy}\times\frac{dy}{dx}\to1\frac{dy}{dx}\to\frac{dy}{dx}$$

The x section gets differentiated as normal

The example below is from 4(c) of the MAS182 exams. In it we are finding the $\frac{dy}{dx}$

$$\cos(xy)+ye^x+x^2=xy^2$$

$$\underbrace{-\sin(xy)\cdot\underbrace{\left[(x)\frac{dy}{dy}\cdot\frac{dy}{dx}+(1)(y)\right]}_{\text{Product rule}}}_{\text{Chain Rule}}+\underbrace{\left[(y)(e^x)+\left(\frac{dy}{dy}\frac{dy}{dx}\right)(e^x)\right]}_{\text{Product Rule}}$$

$$+2x=\underbrace{(x)\left(2y\frac{dy}{dx}\right)+(1x^0)(y)}_{\text{Product Rule}}$$

$$-\sin(xy)(x)\frac{dy}{dx}+y+ye^x+\frac{dy}{dx}e^x+2x=x2y\frac{dy}{dx}+y$$

$$-\sin(xy)(x)\frac{dy}{dx}+\frac{dy}{dx}e^x-x2y\frac{dy}{dx}=y-2x-ye^x-y$$

$$\frac{dy}{dx}\left[-\sin(xy)(x)+e^x-x2y\right]=-2x-ye^x$$

$$\frac{dy}{dx}=\frac{-2x-ye^x}{e^x-\sin(xy)(x)-x2y}$$

Laws of Logarithms – p825

1. $Log_b mn=Log_b m+Log_b n$

   $\ln(mn)=\ln(m)+\ln(n)$

2. $Log_b\frac{m}{n}=\log_b m-\log_b n$ $\qquad \ln(\frac{m}{n})=m-\ln(n)$

3. $Log_b m^n=n\log_b m$ $\qquad n\ln(m)=\ln(m^n)$

4. $Log_b 1=0$ $\qquad \ln(1)=\ln(0)$

5. $Log_b b=1$

6. $Log_a a^y=y$

7. $a^{\log_a x}=x$

Properties Relating e^x and ℓnx $\qquad e^{\ell nx}=x \qquad \ell ne^x=x$

Rule 1 : Derivative of an exponential function – p834 $\frac{d}{dx}(e^x)=e^x$

Rule 2 : Chain Rule for Exponential Functions $\frac{d}{dx}(e^{f(x)})=e^{f(x)}f'(x)$

Rule 3 : Derivative of ℓnx - p845 $\frac{d}{dx}\ell n|x|=\frac{1}{x}$ $\quad (x\neq0)$

Rule 4 : Chain Rule for Logarithmic Functions – p846

If f(x) is a differentiable function then

$$\frac{d}{dx}[\ell nf(x)]=\frac{f'(x)}{f(x)} \qquad [f(x)>0]$$

Continuous Compound Interest

$A=Pe^{rt}$ where

P = Principal

r = Annual rate of interest compounded Annually

t = time in years

A = Accumulated amount at the end of t years

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Natural Log differential

$\frac{d}{dx}e^{f(x)}=f'(x)e^{f(x)}$ if $f(x)=e^{(3x+4)}$ then

$f'(x)=3e^{(3x+4)}$

$\frac{d}{dx}\ell n f(x)=f'(x)\frac{1}{f(x)}$ if $f(x)=\ell n(3x+4)$ then

$f'(x)\frac{3}{3x+4}$

$\Delta=\partial=d$ which all means the change in something.

$$\frac{\Delta y}{\Delta x}=\frac{\delta y}{\partial x}=\frac{dy}{dx}=\frac{The_{chnge_{in_y}}}{when\_'x'\_changes}$$

The second or double derivative. $f''(x)$

The purpose of a derivative is to find the rate of change on an equation. Therefore the the second derivative is to find out the rate of change of the first derivative.

Integration 21.0 $\int_a^b$ The sum of all the areas.

To create each area use height * width as follows.

The function, say x^2 is the height

and the dx is the width

Explained beautifully on page 162 of Principals of Physics by Serway& Jewitt PEC152 Textbook

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Basic integration Rules

Rule 1 : The indefinite integral of a constant

$\int kdx=kx+c$ (k is a constant)

Rule 2 : The Power Rule

$$\int x^n dx=\frac{1}{n+1}x^{n+1}+c \quad (n\neq-1)$$

Rule 3 : The indefinite Integral of a constant Multiple of a function.

$\int c\cdot f(x)dx=c\int f(x)dx$ (c, a constant)

Only a constant can be “moved out”

Eg. $\int x^2dx=\frac{1}{3}x^3+c$

Where as

$x^2\int 1\,dx=x^2(x+c)=x^3+cx^2$

Rule 4 : The sum Rule

$$\int[f(x)\pm g(x)]dx=\int f(x)dx\pm\int g(x)dx$$

Rule 5 : The indefinite integral of the Exponential function p387

$\int e^x dx=e^x+c$ and by applying rule below

$\int e^{-x}dx=-e^{-x}+c$

Tricky example. This is considered a constant $e^{\int\frac{1}{50}dt}=e^{\frac{1}{50}t}=e^{\frac{t}{50}}$

$$\int \ln(x)dx=x(\ln(x)-1)$$

$$\int e^{kx}dx=\frac{1}{k}e^{kx}=\frac{e^{kx}}{k}+c,\quad k\neq0$$

$$\int a^x dx=\frac{a^x}{\ell na}+c$$

$$\int a^{kx}dx=\frac{a^{kx}}{k(\ell na)}+c,\quad k\neq0$$

Rule 6 : The indefinite Integral of the function $f(x)=x^{-1}$ - pg 875

$$\int x^{-1}dx=\int\frac{1}{x}dx=\ell n|x|+c \quad (x\neq0)$$

Rule 5 : The Area between 2 curves

$$\int_a^b[f(x)-g(x)]dx$$

Rule 7: Product Rule for integration - Integration by parts :
Pauls notes – CalcII_Complete.pdf – page 4

$$\int u\,dv = uv-\int v\,du$$

$$\int_a^b udv=uv¿a¿b-\int_a^b vdu$$

D		I
x	+	e^x
1	-	e^x
0	+	e^x

Example :

$$I_{(x)} = \int xe^x dx$$

$$I_{(x)} = +(x)e^x-(1)e^x$$

$$\frac{d}{dx}(\sin kx)=k\cos kx\rightarrow\int\cos kx\,dx=\frac{1}{k}\sin kx+c$$

$$\frac{d}{dx}(\cos kx)=-k\sin kx\rightarrow\int\sin kx\,dx=-\frac{1}{k}\cos kx+c$$

Substitution Method for Integrals (p399 Calc with Apps)

Substitute ‘u’ to be one of the following

1. The quantity under a root or raised to the power
2. The exponent on e
3. The quantity in the denominator

Integrands may need to be rearranged to fit one of these cases.

From www.wolfremalpha.com

Indefinite integrals:

$$\int V\cos(\omega t)\,dt=\frac{V\sin(t\,\omega)}{\omega}+\text{constant}$$

Possible intermediate steps:

$$\int V\cos(t\,\omega)\,dt$$

Factor out constants:

$$=V\int\cos(t\,\omega)\,dt$$

For the integrand $\cos(t\omega)$, substitute $u=t\omega$ and $du=\omega dt$:

$$=\frac{V}{\omega}\int\cos(u)\,du$$

The integral of $\cos(u)$ is $\sin(u)$:

$$=\frac{V\sin(u)}{\omega}+\text{constant}$$

Substitute back for $u=t\omega$:

$$=\frac{V\sin(t\,\omega)}{\omega}+\text{constant}$$

Slope of a Tangent line to the graph f at the point $p(x,f(x))$ - p617

Instantaneous

$$\lim_{h\rightarrow o}\frac{f(x+h)-f(x)}{h}$$

Average

$$\frac{f(x+h)-f(x)}{h}$$

For the Term $5x^6$

5 is the Coefficient
x is the variable
6 is the power
The degree of a polynomial is its highest power.

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$$rational\ expression = \frac{numerator}{denominator}$$

Rational Exponent eg. $a^{\frac{3}{2}}$

Radical Sign $\sqrt{a}$

A common Denominator is needed for addition and subtraction

$$\frac{P}{Q} \pm \frac{R}{S} = \frac{PS}{QS} \pm \frac{RQ}{SQ} = \frac{PS \pm RQ}{QS}$$

Scientific Notation

Eg $1.23 \times 10^{11} = 123.000,000,000$

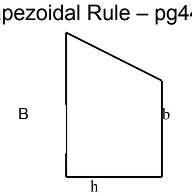
Eg $3.45 \times 10^{-4} = 0.000345$

Exponential is where the power is the variable. Quadratic is where the base is the variable.

Anything Divided by 0 is undefined

Quotient – the result of division. The number of times one quantity is contained in another. Quotients of polynomials are called rational expressions.

Trapezoidal Rule – pg441



Area = $\frac{1}{2}(B+b)h$

$$\frac{e^{ax}}{a} = \int e^{ax}\,dx$$ where a is a constant. – See formulae sheet 6

height

Average value of a function on the interval [a,b]

$$\frac{1}{b-a}\int_a^b f(x)\,dx$$

p573 **Exponential growth and Decay** function. $y = Me^{kx}$ where M = e^c or M = e^{-c}

When doing calculations remember to show M as e^x or ln e if moving it to the other side of the equals sign.

$\frac{dy}{dx}$ means the limit of a small change in y with respect to a small change in x

p575 **Logistic Growth** $\frac{dy}{dx} = k\left(1 - \frac{y}{N}\right)y$ where the $\left(1 - \frac{y}{N}\right)$ is multiplied by k as a limiting factor to reverse the curve.

Logarithm Rule $6\ell nx = \ell nx^{\hat{6}}$

Volume of a cone $V = \frac{1}{3}\pi r^2 h$

Derivatives of trig functions

$D_x(\tan x) = \sec^2 x$

$D_x(\cot x) = -\csc^2 x$

$D_x(\sin x) = \cos x$

Complex Numbers http://www.intmath.com/

A complex number multiplied by it's conjugate is a **real** number.

Ie (3+2j)(3-2j) = **13** → 9 -6j + 6j -4(-1) = **13**

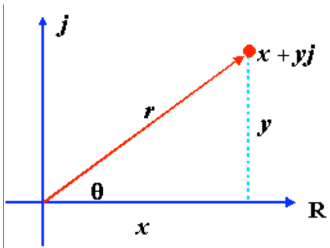
This becomes useful when dividing complex numbers.

There are terms in complex number which mean the same thing. These are stated in the 3 lines below.

r = |z| = mod z = modulus (they are all the same thing)

Θ = theta = 'The argument' = arg z = The argument of mod z = the angle

cis = cos plus i sin = cos plus j sin = cos + i sin = cos + j sin



From Pythagoras, we have: $r^2 = x^2 + y^2$ and basic trigonometry gives us:

$\tan\theta = \frac{y}{x}$ **$x = r\cos\theta$** **$y = r\sin\theta$**

Multiplying the last expression throughout by j gives us:

$yj = jr\sin\theta$

This gives us 2 simultaneous equations

$x = r\cos\theta$ **(1)**

$yj = jr\sin\theta$ **(2)**

Substitute (1) and (2) into the point on the graph

$x+yj = r\cos + jr\sin\theta$

So we can write the **polar form** of a complex number as:

$$\boxed{x + yj = r(\cos\theta + j\sin\theta)}$$

r is the **absolute value** (or **modulus**) of the complex number

θ is the **argument** of the complex number.

There are two other ways of writing the **polar form** of a complex number**:**

r cis θ [means $r(\cos\theta + j\sin\theta)$]

$r < \theta$ [means once again, $r(\cos\theta + j\sin\theta)$]

The **exponential form** of a complex number is:

$$\boxed{r\,e^{j\theta}}$$

(***r*** is the **absolute value** of the complex number, the same as we had before and **θ** is in **radians**.)

Rectangular form	Polar form	Exponential form
$x + yj$ =	$r(\cos\theta + j\sin\theta) = r$ cis θ $= r\angle\theta$	$= re^{j\theta}$ θ
	can be in degrees OR radians θ	MUST be in radians

When performing addition and subtraction of complex numbers, use **rectangular form**. (This is because we just add real parts then add imaginary parts; or subtract real parts, subtract imaginary parts.)

When performing multiplication or finding powers and roots of complex numbers, use **polar** and **exponential forms**. (This is because it is a lot easier than using rectangular form.)

We can generalise the example we just did, as follows:

$$(r_1 e^{\theta_1 j})(r_2 e^{\theta_2 j}) = r_1 r_2 e^{(\theta_1 + \theta_2) j}$$

From this, we can develop a formula for multiplying using polar form:

$$r_1(\cos\theta_1 + j\sin\theta_1) \times r_2(\cos\theta_2 + j\sin\theta_2)$$

$$= r_1 r_2(\cos[\theta_1 + \theta_2] + j\sin[\theta_1 + \theta_2])$$

or with equivalent meaning:

$$r_1\angle\theta_1 \times r_2\angle\theta_2 = r_1 r_2\angle[\theta_1 + \theta_2]$$

In words, all this confusing-looking algebra simply means...

To multiply complex numbers in polar form,

Multiply the ***r*** parts

Add the **angle** parts

To divide complex numbers in polar form,

Divide the ***r*** parts

subtract the **angle** parts

Not in Standard Formulae book

Properties Relating e^x and ℓnx $e^{\ell nx} = x$ $\ell ne^x = x$ eg : $e^{-5\ell nx} = x^{-5}$

DeMoivre's Theorem – for powers and roots of complex numbers

$$[r(\cos\theta + j\sin\theta)]^n = r^n(\cos n\theta + j\sin n\theta)$$

or $(r\angle\theta) = (r^n \angle n\theta)$

General Drawing of polynomials (see sheet 6)

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p429 **Area between curves**

$$\int_a^b [f(x)-g(x)]\,dx$$

trig solutions pMT41

To find ⊠ in degrees $\sin^{-1}\left(\frac{O}{H}\right)=\theta$

To convert degrees to Radians $(x^\circ/180)\pi = Radians$

eg

$$(15^\circ/180)\pi = 0.2617\ Radians$$

To convert Radians to Degrees $x^\circ = \frac{Radians}{\pi}(180^\circ)$

Integration By parts

Rule 3 : Derivative of ℓnx - p845 $\frac{d}{dx}\ell n|x| = \frac{1}{x} \quad (x \neq 0)$

Probability and Stats

$\overline{x}$ = Sample Mean (said "X Bar")

$\overline{\overline{x}}$ = Mean of the mean. See Vee's notes page 34.

s = Sample SD

Factoral = x!

Say 5! (said 5 Factoral) = 5 x 4 x 3 x 2 x 1 =120

0! = 1

On calculator in RUN mode -> OPTN -> Prob -> x!

Permutations – from http://www.intmath.com/Counting-probability/3_Permutations.php

In how many ways can a supermarket manager display 5 brands of cereals in 3 spaces on a shelf?

$$P_3^5 = \frac{5!}{(5-3)!} = \frac{5!}{2!} = 60$$

Combinations – from http://www.intmath.com/Counting-probability/4_Combinations.php

$$C_r^n = \frac{n!}{r!(n-r)!}$$

eg Find the number of ways in which 3 components can be selected from a batch of 20 different components.

$$C_3^{20} = \frac{20!}{3!(20-3)!} = \frac{20!}{3!17!} = 1140$$

http://www.intmath.com/Counting-probability/12_Binomial-probability-distributions.php

The Binomial Probability Distribution

1 .The experiment consists of ***n*** repeated trials;

2. Each trial results in an outcome that may be classified as a success or a failure (hence the name, binomial)

3. The probability of a success, denoted by *p*, remains constant from trial to trial and repeated trials are independent.

$$P(X) = C^n_x p^x q^{n-x}$$

where

n = the number of trials

x = 0, 1, 2, ... n

p = the probability of success in a single trial

q = the probability of failure in a single trial

(i.e. $q = 1 - p$)

C^n_x is a combination

which is the same as $P(x=k) = \binom{n}{k} p^k (1-p)^{n-k}$

where n = number of throws

k = success needed

p = probability per throw of success -eg need 2 to appear 3 times in a dice roll.

To do on the CASIO fx-9860GAU → menu → Stat → F5(Binm) → F1(for Binomial prob Dist) or F2 (for Binomial Cumulative Dist) → Set Data to F2(Var) → enter your values → EXE

Where to use different Probability functions.

1. Binomial Used when there is a definite amount of results that can happen in a definite set of tests.
   Eg : A dice is thrown 100 times. There fore we know that there will be 100 results. Then we talk about the probability of a certain result happening or not.
2. Poison Used when the results are not defined.
   Eg: How many cars on a road in a specific time frame. There is no definite possibility of results like the dice. We would just talk about the chances of more than 1 car per minute, 2 cars per minute, 3 cars per minute and so on.

Poison distribution

$$P(X) = \frac{e^{-\mu}\mu^x}{x!}$$

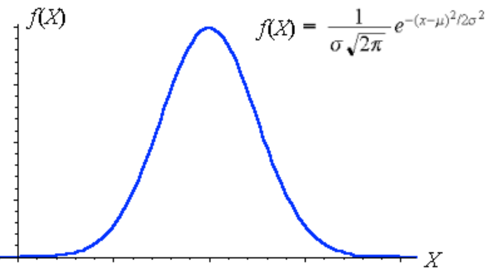
Where e = logarithmic e

μ = Mean

x = 0, 1, 2, ... n

Normal Distribution [**(from intmath.com)**](intmath.com)

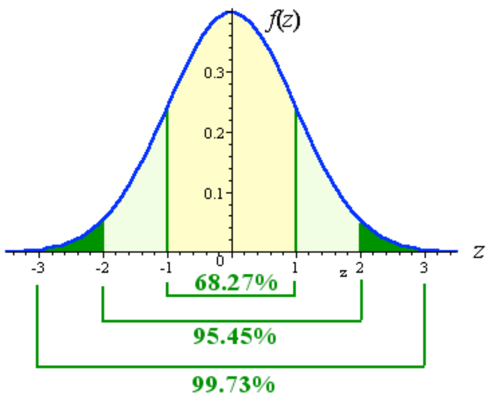
Most distributions in nature occour like this (the bell curve)



The function for this graph is

$$f(X) = \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$$

To make it easier to do calculations we slide it along the graph so the mean (μ) = 0 and the standard deviation (S.D or σ) = 1



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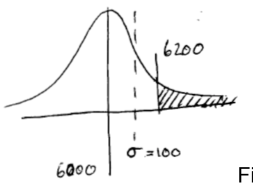
$\sigma = 68.27$
$2\sigma = 95.45$
$3\sigma = 99.73$

To Use the calculator
To calculate the p value between 2 z values.
Eg -1σ and +1σ
MENU → STAT → DIST → NORM → Ncd →
Leave the σ = 1 and the rest is self explanatory.

Standardised Value

(which is the number of σ away from the mean $Z = \frac{x-\mu}{\sigma}$

NB : A t-ratio is like a Z-ratio but using an estimator for σ . t- ratios are not really used in MAS284 and we don't need to worry about them too much.



Find the area to the right of 6200.

This is called the Standardised value. $Z = \frac{6200-6000}{100} = 2$ which is the number of S.D away from the mean.

F-Ratio

$$F = \frac{\text{REG SS/its DF}}{\text{RESIDUAL SS/its DF}} = \frac{\text{REG MS}}{\text{RESIDUAL MS}}$$

What is the purpose of an F-Distribution?

To see whether you can drop variables (indicators) and simplify the model. This is especially useful when you have many variables that you may want to drop.

MRSS → This is the SS over the DF from the minitab output. (taken from Assig3Q2a/b)

Analysis of Variance

Source	DF	SS	MS	F	P
Regression	3	133.71	44.57	4.24	0.045
Residual Error	8	84.02	10.50		
Total	11	217.74			

So in the above example the MRSS would be

$$MRSS = \frac{SS}{DF} = \frac{84.02}{8}$$

$$F = \frac{\frac{RSSDiff}{no.\ of\ predictors\ dropped}}{MRSS\ of\ the\ Full\ Model}$$

or use

$$F = \frac{\text{Positive difference in residual SS/ difference in DF's}}{\text{RSS of Full model / its DF - all } \beta \text{x's(don't forget -1)}}$$

Then we go to the F table and read off it using the top DF for V1 and the bottom DF+1 for V2.
Take the calculated F value and see where it most closed fits in the list of 6 numbers.
Then read across to the left to the P column to find the Probability.
This is the probability that at least one of the predictors is significant. (ie if <= 0.05 then one is probably significant)

F test on calculator
STAT → DIST → F →
Fcd :n:df denominator (top value)
:d:df numerator (lower value)

Standard Deviation of a Set of values.

Say we are looking at how many cars per hour go past on a highway.

1. Over an infinite amount of time you will sometimes get a massive amount of cars pass in a second and other times when there's nothing. So the S.D. will be large.
2. But say we just looked at one 10 hour period. The chances of hitting both extremes is small. So the S.D. will be smaller.
3. The formula to calculate this is as follows. $\sigma(ofaSet) = \frac{\sigma}{\sqrt{n}}$ where n is the set size. In the example above it is 10.
4. When analysing Specification limits we just use a the simple standard deviation and NOT that of a set.

How to find the Standard Deviation on the calculator
MENU → STAT → Type the numbers in the list 1 → CALC (F2) → 1VAR
$\overline{X}$ = mean
For 1 SD use the $x\sigma n-1$ line.
For 2 standard deviations multiply this by 2.

How to find the Standard Deviation

$$\sigma = RMS = \sqrt{npq}$$

Where n = the overall units.
P = # of successes (ie 0.7)
Q = # of failures (ie 0.3)

Another Method of finding the Standard Deviation.

$$\sigma = \frac{\overline{R}}{d_2}$$

See http://easycalculation.com/statistics/learn-standard-deviation.php

$$\sigma = \sqrt{\frac{\sum d^2}{n-1}}$$ is the formula for the SD

Basically, what this says is as follows:

1. Find the deviation "d" for each data point Which is the current value
   d=(X – M)
   M= mean
   X = current value
   n = The total number of values.
2 .Square the value of d (d times itself)
3. Sum (add up) all of the squares
4. Divide the sum by the number of data points (n) minus 1
5. Take the square root of that value

Standard Deviation Method1 Example: To find the Standard deviation of 1,2,3,4,5.

Step 1: Calculate the mean and deviation.

X	M	(X-M)	$(X-M)^2$
1	3	-2	4
2	3	-1	1
3	3	0	0
4	3	1	1
5	3	2	4

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Step 2:Find the sum of $(X-M)^2$
4+1+0+1+4 = 10

Step 3:N = 5, the total number of values.Find N-1.
5-1 = 4

Step 4:Now find Standard Deviation using the formula.
√10/√4 = 1.58113

C-chart **http://en.wikipedia.org/wiki/C-chart** or pg50 of PartA of Vees notes.
Used for typically **total number** of nonconformities per unit.[1] It is also occasionally used to monitor the total number of events occurring in a given unit of time.
-Based on Poison Distribution.
The control limits for this chart type are where is the estimate of the long-term process mean established during control-chart setup.

U chart pg50 of PartA of Vees notes.
Used for typically **average number** of nonconformities per unit.[1]
p-chart is a type of control chart that monitors the proportion of nonconforming units in a sample.

- calculated on the basis of the binomial distribution

control limits found by

$$\bar{p} \pm 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$$

Where $\bar{p}$ = mean success or failure rate and n = the sample size.

$$\bar{p} = \frac{\text{mean success or failure rate}}{\text{number of samples} \times n}$$

What is an np chart? A **p chart** shows the probable values on the y axis and the **np chart** shows the real values on the y axis. Details on how to convert from p to np charts are in the lecture notes partA pg – 47. Or just multiply the p values by n.
I-MR chart Not sure where there is an overview of these charts
When to use different charts (from assignment 2 -4a)

C-chart	:	When figures have a poison distribution
I-MR chart	:	When figures have a normal distribution

Rules for I-MR charts	**K**
1) 1 point more than K standard deviations from center line	3
2) K points in a row on same side of center line	9
3) K points in a row, all increasing or all decreasing	6
4) K points in a row, alternating up and down	14
5) K out of K + 1 points > 2 standard deviations from center line (same side)	2
6) K out of K + 1 points > 1 standard deviation from center line (same side)	4
7) K points in a row within 1 standard deviation of center line (either side)	15
8) K points in a row > 1 standard deviation from center line (either side)	8

These can be varied in Minitab15 by Stat → Control Charts → Variable Charts for individuals → I-MR → I-MR options → Tests

To construct the control limits UCL and LCL for Xbar/R charts

The factors you need for the formulae below are on page 31 of the tables and Formulae book from Murdoch. (ie A_2 D_3 D_4)
See the *mas284FormulaeSheet003b.pdf*

USL = Upper Specification Limit
LSL = Lower Specification Limit
Cp = Capability Potential of a process to perform within the specification range.

$$Cp = \frac{USL - LSL}{6 \times \sigma}$$

Preferably this would be greater than 1 which would mean that no readings are outside the control limits. http://elsmar.com/Cp_vs_Cpk.html

See PartA page 41 of Vee's notes.

$$Cpk = min\left(\frac{USL-\mu}{3\times\sigma}, \frac{\mu-LSL}{3\times\sigma}¿\right)$$

min = means to take the minimum value of the 2 results. If it's =>1 then it is Capable.

Standard Error http://www.ultimacalc.com/html/standard-deviation.html
It can be shown that the standard error of the mean is equal to the standard deviation of the sample divided by the square root of the number of items in that sample.

$$S\tan dardError = SE_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

http://en.wikipedia.org/wiki/Standard_error_(statistics)

H_0 is a null hypothesis
H_1 is an alternate hypothesis
H_2 is the 2nd alternate hypothesis

t-Ratio - $t = \dfrac{\bar{x}-\mu}{s/\sqrt{n}}$

C-charts – page 50 of Part A lecture notes. Add the c and u chart usages to the rule sheet.

Residual
The residual is like the σ in a normal control chart. The residual is just the difference between the estimated line and the actual physical point measured. It is given by the formula ;-

$$s = \sqrt{\frac{\sum residuals^2}{n-2}}$$

residual = observed value – fitted value
Explained on page 2 of PartB of Vee's notes.

The degree of freedom is given by the n. If you have 2 degrees of freedom then n will actually equal 3 because in the above equation n is always calculated by saying n = n -1. Why? Can't remember.

From http://en.wikipedia.org/wiki/Degrees_of_freedom_(statistics)
In statistics, the number of **degrees of freedom** is the number of values in the final calculation of a statistic that are free to vary.[1]

S = Standard Deviation

A t- Distribution is used when taking a sample from a larger pool of data. Usually when taken from this larger pool the mean will stay the same but the SD will be much smaller.

Say you have a 10,000 bits of data and pull out sets of 25. You would say 25 -1 = n. The bigger n is the smaller the SD.

Regression
e_t = Random Error at time t
$y_t = a + b_t + e_t$
$\therefore e_t = y_t - a - b_t$
Generally the regression line is

$$y_t = a + bt$$

where a = y intercept and
b = gradient
RSS = Residual Sum of Squares

Rsq = The coefficient of determination.

To create indicator variables in Minitab Calc → Make indicator Variables.

Reading Minitab printouts
The lower the P value the higher the significance of the line.P values generally above 0.05 (5%) are usually excluded because they are outside 2 standard deviations.

CI = Confidence intervals

c. What is :-
1. Coef
2. SECoef = StDev = Standard Error
3. tsq = time squared
4. T = time
5. F = F-Ratio
6. DF = Degree of Freedom
7. SS = Sum of Squares

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8. MS = SS/DF
9. P = Significance – A high P value means a high residual. We can eliminate results with high P values from minitab print outs. Generally if the significance > 5% (0.05) then we disregard these values. Supposed to be done one at a time and then the minitab output recalculated. (See page 13 Part B of lecture notes)
10. AR =
11. S = Residual Standard Dev S is the residual standard deviation, but how is it calculated? Explained on page 2 of PartB of Vee's notes.
12. Rsq A value of 100% implies a perfect fit.
13. CI = Confidence Interval

~~What is residual error?~~ It is the difference between the observed value and the fitted value.
http://www.null-hypothesis.co.uk/science//item/what_is_a_null_hypothesis
one may either *reject*, or *not reject* the null hypothesis; one cannot *accept* the null hypothesis.

Mathvids.org or .com or something.

Row reducing rules
1. Can add and subtract rows from each other.
2. Can multiply or divide a row by a constant.
3. Can swap rows.

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