$e^{a+b}=e^a \times e^b$

$AX=B$

$¿\Rightarrow A^{-1}AX=A^{-1}B¿\Rightarrow X=A^{-1}B¿$

To find the inverse change $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ to $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Then $A^{-1}=\frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

To find the inverse on a calculator

OPTN Mat **F2** Mat **F1 SHIFT** x^{-1} **)**

On calculator : OPTN → MAT F2 → DET F3

Determinant of order 2 - Pg 822 stew

$\begin{vmatrix} a & b \\ c & d \end{vmatrix}=ad-bc$

Determinant of order 3 - Pg 822 stew

$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}=a_1\begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix}-a_2\begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix}+a_3\begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$

Determining whether a matrix has an inverse Heuv pg 67

If $Det(A)\neq 0$ is a non singular Matrix and inverse exists.

Eigen Values - **Example**:

$B=\begin{bmatrix} 3 & 1 \\ 4 & 0 \end{bmatrix}$ det (b – λI)

$det\begin{bmatrix} 3-\lambda & 1 \\ 4 & 0-\lambda \end{bmatrix}$

= (3 - λ)(-λ) – 4

= λ^2 - 3λ – 4

= (λ + 4) (λ - 1)

Eigenvalues = **λ = 4, λ = -1**

Then to get EigenVectors for each Eigenvalue :-

Insert **λ = 4** into equation above

$\begin{bmatrix} 3-4 & 1 \\ 4 & 0-4 \end{bmatrix}=\begin{bmatrix} -1 & 1 \\ 4 & -4 \end{bmatrix}$

Solve for (b – λI)=0

$\begin{bmatrix} -1 & 1 \\ 4 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=0$

Augment the matrix

$$\left(\begin{array}{cc|c} -1 & 1 & 0 \\ 4 & -4 & 0 \end{array}\right) \quad \textbf{(1.1)}$$

Do row operations on R_2 → $R_2 + 4R_1 = 0$

-1x + 1y = 0 → x = y

Set y = t

So EigenVector for **λ = 4** is $t\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Then substitute **λ = -1** into equation above. Skip to **(1.1)** to get

the Augmented = $\left(\begin{array}{cc|c} 4 & 1 & 0 \\ 4 & 1 & 0 \end{array}\right)$

Do row operations on R_2 → $R_2 - R_1 = 0$

4x + 1y = 0 → x = -1/4y

Set y = t

So EigenVector for **λ = 4** is $t\begin{bmatrix} -\frac{1}{4} \\ 1 \end{bmatrix}$ or multiple of is $t\begin{bmatrix} -1 \\ 4 \end{bmatrix}$

To find EigenValues of 3x3 matrix

$A=\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

Characteristic Equation

$$-\lambda^3+(a+e+i)\lambda^2-\left(\begin{vmatrix} e & f \\ h & i \end{vmatrix}+\begin{vmatrix} a & c \\ g & i \end{vmatrix}+\begin{vmatrix} a & b \\ d & e \end{vmatrix}\right)\lambda+|A$$

Quadratic Formula $f(x)=ax^2+bx+c$

Quadratic solution $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ Can have 0,1 or 2 solutions

If the b^2-4ac section is

–ve =no solution

0 = one solution

+ve = two solutions

Vector definition between 2 points

$\vec{v}=(x_2, y_2, z_2)-(x_1, y_1, z_1)$

Equation for a line in 3D pg 830 Stew

$\vec{r}=\vec{r_0}+t\vec{v}$ where

$\vec{r_0}$ denotes a point on the line.

t Denotes a variable to trace out the line.

$\vec{v}$ Denotes the above vector definition between 2 points

Distance formula in 3 dimensions - pg 803 Stew

$|P_1P_2|=\sqrt{(X_2-X_1)^2+(Y_2-Y_1)^2+(Z_2-Z_1)^2}$

Unit Vector

$$\frac{|P_1P_2|}{\sqrt{(X_2-X_1)^2+(Y_2-Y_1)^2+(Z_2-Z_1)^2}}$$

Equation for a sphere - pg 804 Stew

$(x-h)^2+(y-k)^2+(z-l)^2=r^2$

Dot Product -pg 815 Stew

$a\cdot b=a_1b_1+a_2b_2+a_3b_3$

The resultant is a real number (ie scalar)

Cross Product pg822 Stew

The resultant is a vector which is perpendicular to both the other vectors.

$a\times b=\langle a_2b_3-a_3b_2, a_3b_1-a_1b_3, a_1b_2-a_2b_1, \rangle$

Angle between the vectors a and b – pg 816 Stew

$a\cdot b=|a||b|\cos\theta$

Perpendicular (orthogonal) angles pg 817 Stew

If $a\cdot b=0$ then the two lines are perpendicular.

Orthogonal – 2 lines are orthogonal if they are perpendicular.

The vector $a\times b$ is orthogonal to both a and b.

Parallel vectors 2 vectors are parallel if the cross product **a x b** = 0

Area of a parallelogram The length of a cross product **a x b** is equal to the area of the parallelogram.

Equation of a plane pg 834Stew

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

where **n** = (a,b,c) = normal vector = cross product of 2 vectors on plane and

$r_0=(x_{0,} y_{0,} z_0)$ which is a point on the plane.

Rolle's Theorem There is a value c in (a,b) such that

$f'(c)=\left[\frac{f(b)-f(a)}{b-a}\right]$

Sequences

To find if a sequence is convergant or divergent :

$a_n=\left(\frac{1+n-3n^2}{7+4n^2}\right)$

1. Divide by the highest power of n. ie n^2 and take the limit

$\lim_{n\to\infty} a=\left(\frac{\frac{1}{n^2}+\frac{1}{n}-3}{\frac{7}{n^2}+4}\right) \to \lim_{n\to\infty} a=\left(\frac{0+0-3}{0+4}\right)$

$\to \lim_{n\to\infty} a=\left(\frac{-3}{4}\right)$ Check this rule with Ha?

Series (are a sum of sequences or functions)

Tests of Convergence Ha's hand written notes 6.3

1. Integral Test *(Also used to estimate the sum of a series)*
Basically just do a definite integral for the series.

2. Comparison Test

3. Limit Comparison Test

4. Alternating Series Test

5. Ratio Test. *Ha's typed notes 'Seq. & Series' pg 52*

literally put $a_{(n+1)}$ over a_n and algebraically solve it.

$\lim_{n\to\infty} .\left|\frac{a_{(n+1)}}{a_n}\right|<1$ Absolutely convergent

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$$\lim_{n\to\infty} \left|\frac{a_{(n+1)}}{a_n}\right| > 1 \text{ or } \to \infty$$ Divergent.

$$\lim_{n\to\infty} \left|\frac{a_{(n+1)}}{a_n}\right| = 1$$ Indeterminate → use another method

NB : 1. You will have to divide through by n^x where x is the highest power.

2. Remember to test the end points for convergence.

Power Series 1. A power series may converge or diverge for some values of x

2. The sum of the series is a function where its domain is the set of x where there is convergence.

$$\sum_{n=0}^{\infty} c_n(x-a)^n = c_0 + c_1(x-a) + c_2(x-a) + \ldots..$$

3. This is a power series about a or centred about a.

4. First do the Ratio test on the series at hand.

5. Which should algebraically reduce to the form (x – a).

6. From the Ratio rule we know that for the series to be convergent then -1 < x < 1.So say you get (x – 3) as n → ꝏ :. in this case -1 < |x – 3|. < 1 So just mash x around until it fits this inequality.

7. Then we need to find out whether it converges for 1 and -1 so plug the right numbers in for x an check it.

Taylor Series

1. Take the number of derivatives you need according to the order required. (The order is the power. So if order 4 the do up to the 4th derivative.)

2. The requested Taylor series will be asking for an answer about a number. Say x=0. So solve the original equation and all of the derivatives you've made at x=0 in this case..

3. Note that the original equation is the 0th derivative and this is used in the initial terms.

4. Take these answers and insert them into the Taylor formula.

$$f(x) = \frac{\sum_{n=0}^{\infty} f^n(x_0)}{n!}(x-x_0)^n$$

This is a bit confusing but

f^n is just the result you got at a specific derivative in step **2.**

The x_0 is just the number your solving for. Ie : x=0

$(x-x_0)^n$ Here if you were solving for x=5 and it was step 3 then you would leave it at . See Paul's notes for more detail.

L'Hospital's Rule Ha's printed notes 'Opening remarks' pg14

When we are taking the limits (ie where a curve crosses another line) we often get to the point where we can't get an answer because we get 0/0 at a limit.

Eg : $$\lim_{x\to 2} a = \left(\frac{x^2-4}{x-2}\right) = \left(\frac{0}{0}\right)$$

To resolve this we take the differential of both the numerator and the denominator.

$$\lim_{x\to 2} a = \left(\frac{2x}{1}\right) = 4$$

Eulers Methd pg35 Ha's notes -ODE's

$$\frac{dy}{dt} \approx \frac{y_{k+1}-y_k}{\Delta t} = y_k t_k, k = 0,1,2,3,\ldots$$

Rearrange to $y_{k+1} = y_k + \Delta t(y_k k \Delta t)$

Newtons Method pg269 Stew

1. Plot the graph and find a point close to the intersect to get a starting point which will be x_1
2. Rearrange to the standard form. Ie $x^6 - 2 = 0$
3. Apply $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$
4. Apply over and over to get a series of numbers $x_1, x_2, x_3, x_4, \ldots$

When 2 of these terms are the same to 8dp then you have your answer.

P572 Separable Differential Equations – Means to be able to get one variable on one side like you normally try to do in an algebraic equation.

$\dfrac{dy}{dx} = \dfrac{f(x)}{g(y)}$ is resolved by cross multiplying $g(y)dy = f(x)dx$

then integrating both sides $\int g(y)dy = \int f(x)dx$

P584 **Solving a Linear First Order Differential Equation**

1. Put the equation in the linear form $\dfrac{dy}{dx} + P(x)y = Q(x)$
2. Find the integrating factor $I(x) = e^{\int P(x)dx}$
3. Multiply each term of the equation from Step 1. by $I(x)$.
4. Replace the sum of the terms on the left with $(I(x)y)'$ .This is because with the interating factor they are nothing more than the product rule.
5. Integrate both sides of the equation.
6. Solve for y.

2nd Order DE's – Homogenous y_H = homogeneous soln.

A second order DE is one that has a 2nd derivative in it defined by either y'' or $\dfrac{dy^2}{d^2x}$

1. Try to rearrange to the format y''-y'+y=0
2. Let $y = e^{\lambda t}$ and $y' = \lambda e^{\lambda t}$ and $y'' = \lambda^2 e^{\lambda t}$
3. Substitute the above values in.
4. Rearrange to be $e^{\lambda t}(\lambda^2 - \lambda + 1) = 0$
5. Calculated the inside of the brackets as a quadratic.

a. Two real roots $\lambda_1 \lambda_2$

$$y_H = Ae^{\lambda_1 x} + Be^{\lambda_2 x}$$

b. Complex conjugate roots $\lambda = \alpha \pm i\beta$

$$y_H = e^{\alpha x}(C\cos\beta x + D\sin\beta x)$$

Then insert your initial values to find C & D. You may have to take the derivative of the answer to find C or D.

c. Single real root λ

$$y_H = (Ax + B)e^{\lambda x}$$

Partial Fractional Expansions

1. Break up polynomial on bottom into it's individual roots (poles). - If we can not because it is an answer of complex form we need to use the procedure called 'Completing the square'

2. Above each pole in the position of the numerator write A,B,C......etc.

3. Each of these terms are now in the form..

$$\frac{x^2+2x-1}{x(2x-1)(x+2)} = \frac{A}{x} + \frac{B}{2x-1} + \frac{C}{x+2}$$

4. Create the common denominator for additions.

$$x^2+2x-1 = A(2x-1)(x+2) + B(x)(x+2) + C(x)(2x-1)$$

Then mash it around to look like :

$$x^2+2x-1 = (2A+B+2C)x^2 + (3A+2B-C)x - 2A$$

From here you have 3 simultaneous eqations

$$2A+B+2C=1$$
$$3A+2B-C=2$$
$$-2A \quad .=-1$$

5. From here we have 2 different ways we can solve this.

a. By using an inverse matrix solution or

b. by substituting in a number for x to cancel out either A,B or C. This will need to be done 3 times.

Rule 7 Product Rule for integration - Integration by parts :

$$\int u\,dv = uv - \int v\,du \qquad \int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

D		I
x	+	e^x
1	-	e^x
0	+	e^x

Example :

$I_{(x)} = \int xe^x\,dx \quad \to \quad I_{(x)} = +(x)e^x - (1)e^x$

Example of complex manipulation :

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$z^3 = -2 + 2i$

$z^3 = 2\sqrt{2} \arctan > -2/2 \rightarrow$ change to $r e^{\theta i}$ form

$z^3 = 2\sqrt{2}\, e^{\arctan -1 + 2k\pi}$ where $k = 0,1,2,3\ldots$

$z_1 = (2\sqrt{2}\, e^{\arctan -1 + 2k\pi})^{\frac{1}{3}}$

$z_2 = (2\sqrt{2}\, e^{\arctan -1 + 2k\pi})^{\frac{2}{3}}$

$z_3 = (2\sqrt{2}\, e^{\arctan -1 + 2k\pi})^{\frac{3}{3}}$

Row reducing rules

1. Can add and subtract rows from each other.
2. Can multiply or divide a row by a constant.
3. Can swap rows.

To check for Saddle Point – Take det. At each point of extrema

$Det(a,b) \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix} = f_{xx} f_{yy} - f_{xy}^2$ If Det(a,b) < 0 Saddle

Completing the Square Best to look at the examples in Pauls notes here but this is a brief description of the format.

$x^2 + bx = x$ Is the general format of a quadratic. Add $\left(\frac{b}{2}\right)^2$ to get a factorable quadratic. $x^2 + bx + \left(\frac{b}{2}\right)^2 = x + \left(\frac{b}{2}\right)^2$

Angles using a 3,4,5 triangle.

Degrees	**0°**	**30°**	**45°**	**60°**	**90°**
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
Sin θ	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$
Cos θ	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$
Tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	**DN θ**

Compare what you have to the formula sheet attached to the exam

thus $G(y) = F(x)$ where the G an F represent the anti-derivatives of g and f.

When finding Extrema of multi-variable functions

– like single variable
- set each derivative to =0
- factor out each variable
- find all values of each variable for when each derivative =0
- treat as simultaneous equations to make easier.

Possibles to add to rule sheet :
Ex06 Q5a and b

Ex 06 Question 6
Taylor series and convergence rules.

Determinant and linear independence

Inverse of a 2*2 matrix – Larger Matricis done by calculator

Identity = $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ The purpose of an inverse matrix is to cancel itself out when solving an equation of matrices

Calculator : OPTN → F2 MAT → F6 → F1 Iden → size of matrix → F6 → F1 MAT → ALPHA → choose letter.

Matrix written as (Row, Column)
Eg. Size of a Multiplication of matrices (3 X 2) and (2 x 4)
The columns in A must equal the rows in B
So the above can multiply and the result is a (3 X 4) matrix.
For $A^{-1} = 0$ the ad – bc section above must = 0

Matrix example

$$\begin{bmatrix} 3 & 0 & 1 \\ 1 & -2 & 0 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ 0 & -1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 8 \\ -1 & 4 \end{bmatrix}$$

Multiply top row of A by first column of B. Then by second column of B.
Then bottom row of A by first Column of B. Then second column of B.

Ha's hand written notes
systems_of_linear_equations_ha_hand_written_notes.pdf

- We then call **x** an eigenvector of A and **λ** an eigenvalue of A.
  $(\lambda I - A)x = 0$ pg 315 pauls.
- Will need long division of polynomials – Pauls notes Alg_Complete.pdf pg245

-Definition 4 The set of all solutions to (λ I − A) x = 0 is called the eigenspace of A corresponding to λ .

Quadratic Second Rule : pg 161L – 10
eg:

$x^2 - 6x + 15$

$x^2 + 2ab + b^2$ where a=x
b = -3
Will have left over 15 - 9 = 6

Therefore $(x-3)^2 + 6$

Maybe put examples of 2003q5 into cheat sheet????

$z^3 = 1 + i$

$z^3 = \sqrt{2} \angle \frac{\pi}{4}$

$z = \left(\sqrt{2} \angle \frac{\pi}{4}\right)^{\frac{1}{3}}$

Also expressed as $re^{i\theta}$

$z = \left[2^{\frac{1}{2}} e^{i\left(\frac{\pi}{4} + 2k\pi\right)}\right]^{\frac{1}{3}}$ where k=0,1,2..n

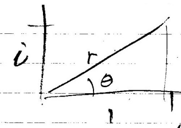
$z = 2^{\frac{1}{6}} e^{i\left(\frac{\pi}{12} + \frac{2k\pi}{3}\right)}$

Convert back to a + ib

$z = 2^{\frac{1}{6}} e^{i\frac{\pi}{12}}$

$= 2^{\frac{1}{6}} \angle \frac{\pi}{12}$

$= 2^{\frac{1}{6}} \left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12}\right)$



$\tan \theta = \frac{i}{1}$

$\theta = \arctan i$

$\theta = \frac{\pi}{4}$

$\cos \frac{\pi}{4} = \frac{1}{r}$

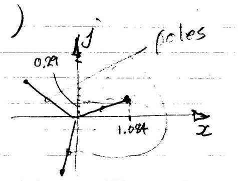
$r = \frac{1}{\cos \frac{\pi}{4}}$

$r = 1.414 = \sqrt{2}$

(A1) $= 2^{\frac{1}{6}} \left(\cos \frac{\pi}{12} + i \sin \frac{\pi}{12}\right)$

$x = 2^{\frac{1}{6}} \cos \frac{\pi}{12} = 1.084$

$y = j 2^{\frac{1}{6}} \sin \frac{\pi}{12} = j0.2905$

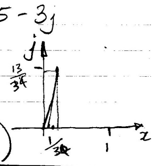


$z = x + jb \Rightarrow z = 1.084 + j0.2905$

$(5+3j)z = 2i - 1 \Rightarrow z = \frac{-1+2j}{5+3j} \cdot \frac{5-3j}{5-3j}$

$\Rightarrow \frac{-5+3j+10j+6}{25-15j+15j+9} \Rightarrow z = \frac{1+13j}{34}$

$\Rightarrow z = \frac{1}{34}(1+13j)$



So 3 answers are (for 3 different poles)

(A2) $2^{\frac{1}{6}} \left(\cos \frac{\pi}{12} + \frac{2\pi}{3} + i \sin \frac{\pi}{12} + \frac{2\pi}{3}\right)$

(A3) " " $+ 2\left(\frac{2\pi}{3}\right)$ ... " " $2\left(\frac{2\pi}{3}\right)$

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