References

ENG125/243 Txtbk → Electrical Engineering – Fourth Ed. By Allan R. Hambley

Other reference → Alexander, Fundamentals of Electric Circuits, 3rd Edition. McGraw Hill, 2007.

Calculator used → Casio fx – 9860G AU

Calculator book → Mathematics with a graphics calculator - Casio fx – 9860G Auby Barry Kissane & Marian Kemp

Lecturer 348 → Beyris

Terms :

Slack (Swing) bus : ENG348 Lect14 p3
Only one per system
Takes up the slack providing the power requirements to satisfy the conditions at the remaining buses.

WYE circuits

Line to line voltages – pg 248 Hamb

$$V_L = \sqrt{3} V_Y \qquad (5.96)$$

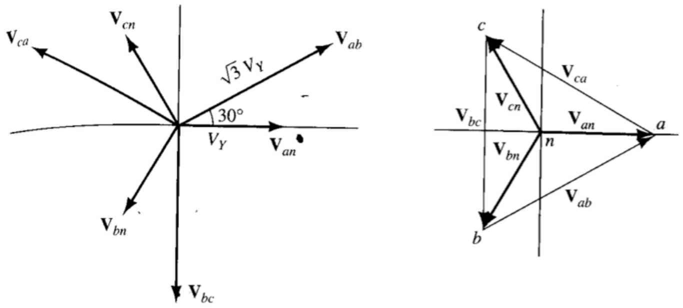
$$V_{ab} = V_{an} \times \sqrt{3} /\_30^o \qquad (5.97)$$

The line to line voltage is $\sqrt{3}$ x the line to neutral voltage.

This is only for positive sequence phasors. For negative sequence use

$$V_{ab} = V_{an} \times \sqrt{3} < -30^o$$

Add diagram here from WSAC-19 or page 249. hambley.



(a) All phasors starting from the origin

(b) A more intuitive way to draw the phasor diagram

Wye to Delta load conversion hamb pg 251

$$Z_\Delta = 3 Z_Y \qquad (5.100)$$

$$V_L = \sqrt{3} V_Y \ /\_30^o$$

(5.96) pg 248

Phase Voltage = Line to Neutral Voltage – p245 Hamb

Per unit calculations – ENG347 P7L3

S_{base} is continuous across the whole circuit.

$$V_{pu} = \frac{V}{V_{base}} \qquad I_{pu} = \frac{I}{I_{base}} \qquad Z_{base} = \frac{(V_{base})^2}{S_{base}}$$

$$I_{base} = \frac{S_{base}}{V_{base}} \qquad S_{pu} = V_{pu} \times I^*_{pu} \qquad S_{pu} = \frac{S}{S_{base}}$$

$$X_{pu} = \frac{X}{Z_{base}} \qquad Z_{pu} = \frac{Z}{Z_{base}} \qquad Z_{pu} = \frac{V_{pu}}{I_{pu}}$$

$$R_{pu} = \frac{R}{Z_{base}} \qquad P_{pu} = |I_{pu}|^2 \times R_{pu}$$

Fault Level pu : ENG348 Lect18 p9

$$FL_{pu} = \frac{FL}{S_{base}}$$

where

$$FL = \sqrt{3} V_{nominated} I_f$$

Permeability of free space

$$\epsilon = \epsilon_0 = 8.85 \times 10^{-12} \ F/m$$

Omega :

$$\omega = 2\pi f$$

$$\omega = 2\pi 50\text{Hz} = 314.1592654$$

$$\omega = 2\pi 60\text{Hz} = 376.9911184$$

$$1000 \ mils = 1 \ inch$$

$$10^6 \ sq \ mils = 1 \ sq \ inch$$

$$\frac{\pi}{4} \ circular \ mils \ (cmils) = 1 \ sq \ mil$$

$$\frac{\pi}{4} \text{x } 10^6 \ cmils = 1 \ sq \ inch$$

$$0.15500 \ sq \ inches = 1 \ sq \ cm$$

$$0.39370 \ inches = 1 \ cm$$

$$5280 \ ft = 1 \ mile \ (mi)$$

$$0.621371 \ mi = 1 \ km$$

$$\frac{1}{\frac{1}{2}+\frac{1}{3}} = \frac{2\times3\times1}{\frac{3\times1}{3\times2}+\frac{2\times1}{3\times3}} = \frac{2\times3}{\frac{3}{6}+\frac{2}{6}} = \frac{2\times3}{3+2}$$

Power – P35 PEC152 55.3 $P = i^2 R$

55.4 $P = v^2 / R$

55.5 $P = Vi$

Average Power through a resistance based on voltage hamb pg 202

$$P_{avg} = \frac{V^2_{rms}}{R}$$

Average Power through a resistance based on current pg 202

$$P_{avg} = I^2_{rms} R$$

Average power Calculation hamb pg228

$P = V_{rms} I_{rms} \cos(\theta)$ **or** $P_{avg} = I^2_{rms} R$ **pg 202**

P35 **Calculation of Power**

$$P = i^2 R \qquad P = \frac{v^2}{R} \qquad P = Vi$$

All complex power components in a circuit will add to zero. *This is whether they are Series or Parallel.*

$P_1 + P_2 + P_3 = 0$

Complex Power – **ENG348 Lect 1 p6**

$S = VI^*$ Where I^* = the complex conjugate

or in extension to this (Lect 1 pg 16

$$S = VI^* = V\left(\frac{V^*}{Z^*}\right) = V\left(\frac{|V^*|^2}{Z^*}\right)$$

Power Factor Triangle - Hambley pg228 - 230

Q = The power within the circuit generally held by inductance and Capacitance.

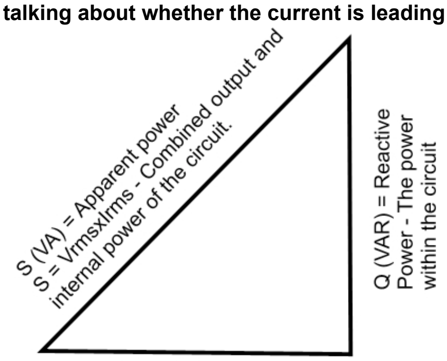
Q is called reactive power. It has the unit of var, rather than watt. It is positive for inductive elements,when current lags voltage ($\theta I < \theta V$); and negative for capacitive elements when current leads voltage.

P = The real or **active** power output by the circuit without Q.

S = Is the apparent power which is a combination of Q and P using trig calculations.

The voltage is the thing changing angle generally whilst the current stays at 0°.

We are talking about whether the current is leading or lagging.



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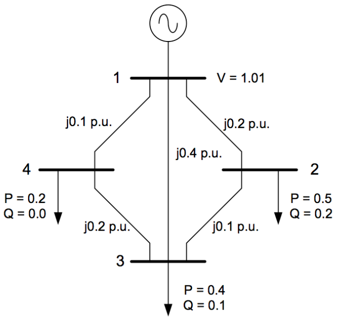
KCL and Complex Power - ENG348 SG15 Q4

As all currents flowing into a point = 0 in KCL, but all complex powers in a circuit add to 0. Or in this case all power flowing into a node must equal 0.
At bus 2 in the following diagram we can write :

$$0 = S_{12} - S_{out} + S_{32}$$



Complex Power on a Capacitor – Gregs email 13/08/2010

The definition for complex power is S = VI*, so the definition for complex power dissipated in the capacitor is

Sc = Vc Ic*

where Vc is the phasor voltage across the capacitor and Ic is the phasor current through the capacitor. As there is no real power dissipated in a capacitor then
the complex power only has an imaginary component, so that

Sc = jQc = VIc*.

In this case Qc is a negative number. However, on page 21 (Lect 01), I have
set Qc to be a positive number, so the negative sign is used so that the reactive power is negative.

This appendix was part of last year's notes. This year I am adopting the convention that Qc will be a negative number, so I would use jQc, where Qc is negative, rather than -jQc where Qc is positive. The result is the same.

From Lect 1 p14

Complex power:	$S = P + jQ$	
Active power:	$P = VI\cos(\phi_v - \phi_i)$	unit: watt (W)
Reactive power:	$Q = VI\sin(\phi_v - \phi_i)$	unit: VAr
Apparent power:	$\lvert S \rvert = VI$	unit: VA

Complex Current : Lect 3 p5

$$S_L = \mathbf{V}_L \mathbf{I}^* = P_L + jQ_L$$

$$\therefore \quad \mathbf{I}^* = \frac{P_L + jQ_L}{\mathbf{V}_L}$$

$$\therefore \quad \mathbf{I} = \frac{P_L - jQ_L}{\mathbf{V}_L^*}$$

Another way of finding Complex Power using ohms Law : pg 348SG10-9

$$S = VI^*$$

$$S = V \times \left(\frac{V}{Z}\right)^*$$

$$S = V \times \left(\frac{V}{\frac{1}{j\omega C}}\right)^*$$

and if finding a Capacitance use the complex

$$S = V \times \left(\frac{V}{1} \times \frac{j\omega C}{1}\right)^*$$

$$S = V \times (V \times -j\omega C)$$

$$S = -j\omega C V^2$$

only :

$$Q = -\omega C V^2$$

Power delivered to a Bus from a Synchronous Generator ENG348 SG7 Q5 :

$$P = \frac{E \times V_{bus}}{X} \times \sin\delta \quad \textbf{or} \quad P = \frac{E/\_\delta \times V_{bus}/\_0}{X} \times \sin\delta$$

and the reactive power is

$$Q_{12} = \frac{V_1 V_2 \cos\delta - V_2^2}{X}$$

therefore the total power is

$$S_{12} = P_{12} + jQ_{12}$$

Synchronous Generators

When the power specification is given, it is usually for all 3 phases combined.

Mechanical power delivered by the prime mover : ENG348 Lect 7 p4

$$P_{¿} = \tau_{\text{applied}} \times \omega_m \qquad (4)$$

Power converted to electrical power : **ENG348 Lect 7 p4**

$$P_{\text{conv}} = \tau_{ind}\omega_m = 3E_A I_A \cos\gamma \qquad (5)$$

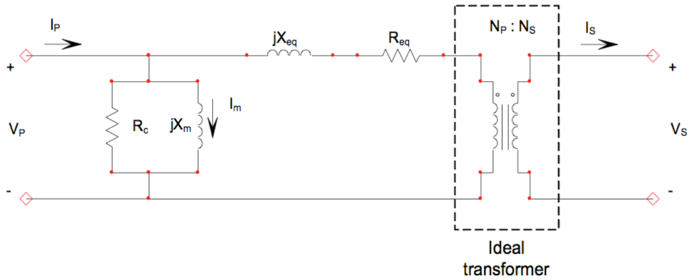
Where γ is the angle between the induced voltage and the current phasors

Actual power on a balanced 3 phase load is

$$P_{\text{out}} = 3V_A I_A \cos\theta \qquad (6)$$

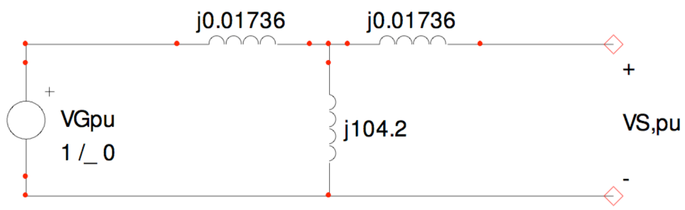
Transformers : ENG348 Lect4 page 3 & SG5 Q5

Copper losses, Core losses and magnetising inductances can be modelled in Parallel to the core windings.



Leakage is modelled in series as normal but split either side of the **magnetising reactance**. The example below is from ENG348 SG5 Q5.

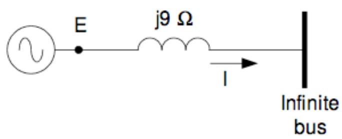
The leakage $X_{lpu} = 0.03472$ is split either side of the magnetising reactance j104.2. Ie $X_{lpu} = \frac{0.03472}{2} = 0.01736$



ENG348 Lect 7 p6

W = Active power : Controlled by prime mover
X = Reactive power : Controlled by the field current.

Power to an infinite Bus Bar : ENG348 lect 7 p5



The power to an infinite Bus Bar remains constant no matter what the generator does.
The infinite bus bar models the entire power grid and thus the generator has little effect on it's voltage.

Power Angle : Page 11 lecture 7

Was derived in Lecture 01. When there is real power though it'll need to be rederived as in Q2 workshop 3. Or derived on Lect7 p8

$$P_L = P_G = \frac{E_A V_x \sin\delta}{X}$$

where δ is the angle of E_A

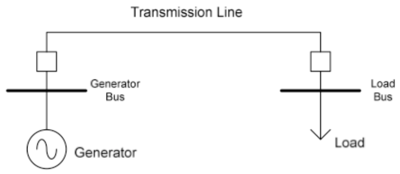
Core Flux vs RMS Voltage : Lect 5 page 8

$$\phi_{\max} = \frac{\sqrt{2}\, V_{in,rms}}{2\pi\, f\, N_P} = \frac{V_{in,rms}}{\sqrt{2}\pi\, f\, N_P}$$

Instantaneous and Real Power calculations :

Instantaneous power is to be calculated keeping time (t) in the equation. Real power is calculated using phasors and is converted into rms values by dividing by $\sqrt{2}$.

2. The single-line diagram for a simple two-bus system is shown in the figure below. The load bus absorbs real and reactive powers of 2.8 and 0.6 per unit. The voltage at the generator bus is 1.0 per unit. The line impedance is j0.04 per unit. Find the voltage at the load bus, and the power supplied by the generator.



Maybe add SG3 q4 to rule sheet for exam.

From KVL: V ... (j0.04) I

$$V_L^{(4)} = 1.0 - \frac{0.1145\angle 77.9^\circ}{V_L^{(3)*}} = 1.0 - \frac{0.1145\angle 77.9^\circ}{0.968\angle +6.64^\circ}$$

$$= 0.962 - j\,0.112$$

$$= 0.968\angle -6.64^\circ$$

The process has clearly converged to the solution: $V_L = 0.968\angle -6.64^\circ \ pu$

The power supplied by the generator is given by

$$S_G = V_G I^*$$

$$= V_G \times \left(\frac{V_G - V_L}{j0.04}\right)^* = 1.0 \times \left(\frac{1.0 - (0.962 - j0.112)}{j0.04}\right)^*$$

$$= \left(\frac{0.038 + j0.112}{j\,0.04}\right)^*$$

0°. Then

$$= 2.8 + j0.95$$

$$= 0.976 - j0.112$$

$$= 0.982\angle -6.55^\circ$$

At the next iteration:

$$V_L^{(2)} = 1.0 - \frac{0.1145\angle 77.9^\circ}{V_L^{(1)*}} = 1.0 - \frac{0.1145\angle 77.9^\circ}{0.982\angle +6.55^\circ}$$

$$= 0.963 - j\,0.110$$

$$= 0.969\angle -6.52^\circ$$

At the next iteration:

$$V_L^{(3)} = 1.0 - \frac{0.1145\angle 77.9^\circ}{V_L^{(2)*}} = 1.0 - \frac{0.1145\angle 77.9^\circ}{0.969\angle +6.52^\circ}$$

$$= 0.962 - j\,0.112$$

$$= 0.968\angle -6.64^\circ$$

At the next iteration:

Delta to Y connected Transformers : ENG348 Lect 6 page 6
Where one side of the transformer is physically connected to a Y setup and the other side is physically connected to a Delta setup.

Therefore the turns ratio becomes :

$$\frac{N_\Delta}{N_Y\sqrt{3}}$$

ENG347 Lect 10 p6 (Example Lect 10 p8)
Equivalencies on either side of a transformer
To remove a transformer out to simplify things we take the loads on one side and do this ratio multipliaction.

$$Z'_P = \left(\frac{N_P}{N_S}\right)^2 Z_S$$ where $_P$ denotes Primary and $_S$ denotes Secondary.

Power Factor values :
arcos 0.95 = 18.195
arcos 0.90 = 25.842
arcos 0.80 = 36.869
arcos 0.70 = 45.570
arcos 0.60 = 53.130
arcos 0.50 = 60.000

Omega Values :

$$\omega = 2\pi f$$

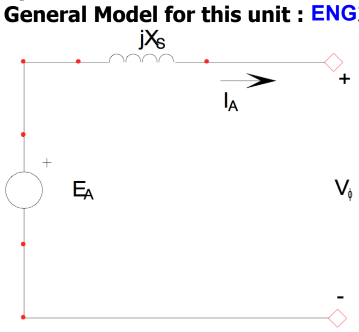
$$\omega = 2\pi 50 = 314.16$$

$$\omega = 2\pi 60 = 376.99$$

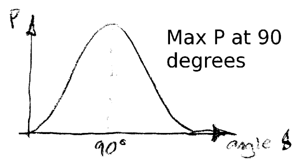
SIL - Surge Impedance Loading – Lesson 13 page 2
In a long line (say 10's or 100's of km) (Greg was saying say more than 500km long) we will get a wave of values happening. So to bring the line back to neutral we can place a series of inductors and capacitors along the line to keep it level along the line. This is the equivalent of bringing the phase back into unity (0 degrees) as we have done with previous equations.

Also if the load impedance is not the same as the SIL then there will be a reflected signal back down the line. We can put an impedance at the termination of the line to stop this reflection (used in communications) but in power engineering we can't control the load impedance so we use the SIL value to just insert capacitors and inductors at various places on the line.

Synchronous Generators :
General Model for this unit : ENG348 lect7 page 3



Excitation ENG348 SG7 Q5 : Excitation is the Generator source voltage. Maximum power will be when this voltage is at 90°



Power delivered to a Bus from a Synchronous Generator ENG348 SG7 Q5 :

$$P = \frac{E \times V_{bus}}{X} \times \sin\delta$$

Current of Synchronous Generator ENG348 SG7 Q5 **:**

$$I_A = \frac{E_{Excitation} - V_{bus}}{jX}$$ **or**

$$I_A = \frac{E_{Excitation}\angle\delta - V_{bus}\angle 0}{jX}$$

Voltage and Current on either side of a transformer : ENG347 Lect10 page 4

$$\frac{V_P}{V_S} = \frac{N_P}{N_S} \qquad \frac{I_P}{I_S} = \frac{N_S}{N_P}$$

To transfer the impedance on one side of a transformer to the other side : ENG347 Lect10 page 6

$$R_{HV} = \left(\frac{N1}{N2}\right)^2 \times R_{LV}$$

Maximum power delivered to a Bus from a Synchronous Generator SG7 Q5 :
Stator windings are also called the Armature Windings in a Synchronous Generator. ENG348 Lect 7 p1

3 phase windings in a Synchronous Generator : ENG348 lect 7 p1

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$$e_{aa'}(t) = \sqrt{2}\,E_A \cos(\omega_e t)$$
$$e_{bb'}(t) = \sqrt{2}\,E_A \cos(\omega_e t - 120°)$$
$$e_{cc'}(t) = \sqrt{2}\,E_A \cos(\omega_e t - 240°)$$

Associated Synchronous Reactance to the inductance L_S **in the air gap :** ENG348 lect7 p2

$$X_s = 2\pi f_e L_S$$

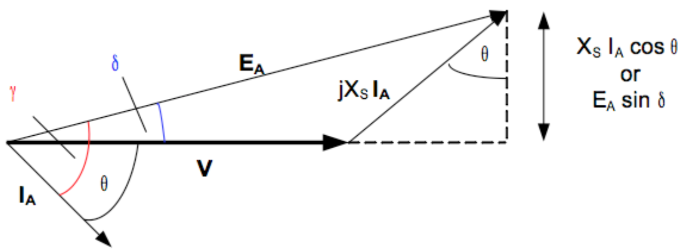
Mechanical power delivered by the prime mover : ENG348 lect 7 p4

$$P_{mech} = \tau_{\text{applied}}\,\omega_m$$

Induced Torque which is left after power losses due to friction, windage and core losses are taken into account. This left over is converted into electrical power: ENG347 lect 7 p4

$$P_{\text{conv}} = \tau_{\text{ind}}\,\omega_m = 3E_A I_A \cos\gamma$$

where γ is the angle between induced voltage and current phasors
Diagram from eng348 lect 7 p5

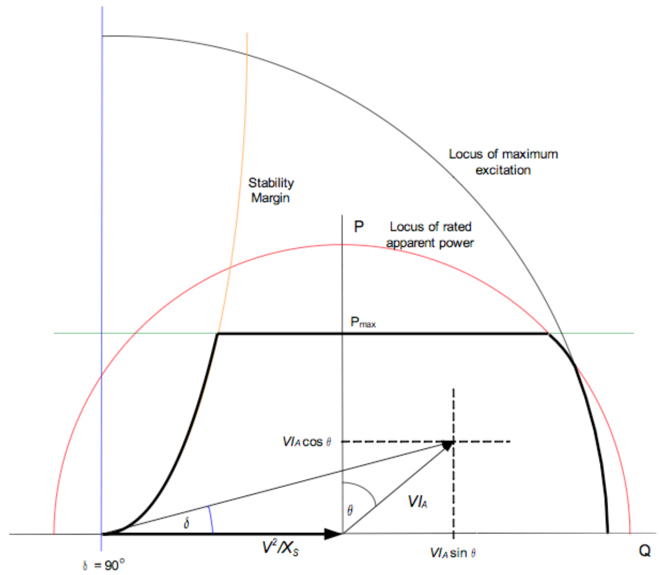


δ is also known as the Torque angle, load angle or the power angle. Typical full load power angles are around 15 -20°.

Eng348 Lect7 p7

$$E_A = jX_S I_A + V$$

Capability Curve or Performance chart or Operating Chart.



Generator operation : At the tip of VI_A
Stability Margin : A 10% reduction for each setting of E_A .
Locus of rated apparent power : Created from VI_A
PMAX : Is the maximum power the turbine can output.
Max Power Angle (the blue line) : is because max power happens at 90°
Locus of maximum excitation : Usually defined by the knee of the magnetisation curve.

Heat balance equation of transmission lines : ENG348 lect9 p5
where

$$I^2 R_{20}\{1 + \alpha(t + \theta)\}$$
$$= 387(V.d)^{0.448}\theta + \pi.E_C.s.d\{(t + \theta + 273)^4 - (t + 273)^4\} - \alpha_S.S.d$$

I = current rating in amps
R_{20} = resistance of conductor at 20°C
α = temperature coefficient of resistance per °C
t = ambient temperature in °C
θ = temperature rise
α_S = solar absorption coefficient
S = intensity of solar radiation, watts/m²
d = conductor diameter in mm
V = wind velocity normal to conductor in m/s
E_C = emissivity of conductor
s = ***Stefan-Boltzmann's constant*** = 5.7×10^{-8} watts/m²

Power transfer over a transmission line : ENG348 lect9 p5

$$\frac{S}{3} = \frac{V_{LL}}{\sqrt{3}} I_L$$

Corona discharge : ENG348 lect9 p6
Usually happens when electric field goes above 18 kV/cm.

DC line resistance of a conducting material: ENG348 lect9 pg8

$$R_{dc}(T) = \frac{\rho(T)I}{A}\Omega$$

Where $\rho(T)$ is the conductors **resistivity** at temperature (T)

Actual conductor length due to strand twisting : ENG348 lect9 pg 8
I(stand) = 1.02 x I(cable)

Variance in resistivity due to Temperature : ENG348 lect9 pg 8

$$\rho(T_2) = \rho(T_1)\left(\frac{T_2 + T}{T_1 + T}\right)$$

AC resistance in a conductor : ENG348 lect9 pg 10

$$R_{ac} = \frac{P_{loss}}{I_{rms}^{\ 2}}\ \Omega$$

Inductance in Transmission lines : **ENG348 lect10 pg 1**
An excellent rundown of how the following equations are put together are shown in the beginning of this lecture. Mainly built on things learnt in ENG347.

Inductance in Transmission lines : ENG348 lect10 pg 3

$$L_{\text{TP,I}} = \frac{\mu_0}{8\pi} + \frac{\mu_0}{2\pi}\ln\left(\frac{R_P}{R}\right) \quad H/m$$

where the first term $\frac{\mu_0}{8\pi}$ is the internal inductance and the second term is the external inductance.

An easier combination of the 2 is below

$$L_{\text{TP,I}} = \frac{\mu_0}{2\pi}\ln\left(\frac{R_P}{R'}\right) \quad H/m$$

where

$$R' = e^{\frac{-1}{4}} R = 0.7788\,R$$

R' is the effectiver Radius of the wire. AND R is the radius.

L is also known as the **Reactance** or $X_L = \omega L$ in units of Ω/km
ENG348 lect12 pg2 where $2\pi f = \omega$

Y is also known as the **Susceptance** or $B = \omega C$ in units of S/km
ENG348 lect12 pg3

Converting a stranded core radius to the equivalent solid radius :
eng348 lect10 pg4

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$$r_{solid} = \sqrt{\frac{A_{stranded}}{\pi}}$$

Composite Conductors : ENG348 lect10 pg 7
Stranded lines have an equivalency to solid lines. This is called :
GMR (Geometric Mean Radius)

$$GMR = \sqrt[N^2]{\prod_{k=1}^{N}\prod_{m=1}^{N} D_{km}}$$

or from eng348 lect10 pg 11

$$GMR = \sqrt[4]{(R')^2 D^2}$$

Distance between lines is known as the
GMD (Geometric Mean Distance)

$$GMD = \sqrt[NM]{\prod_{k=1}^{N}\prod_{m=1}^{M} D_{km}}$$ and for 3phase $$GMD = \sqrt[3]{D_{ab}\,D_{bc}\,D_{ca}}$$

The second formula is also used when spacing is **NOT** equal. See eng348 Lect11 p10.

Inductance for Single phase 2 wire line : ENG348 lect10 pg7

$$L = 2 \times 10^{-7}\,\ell n \frac{GMD}{GMR} \quad H/m \qquad (14)$$

Bundled Conductors : ENG348 lect10 pg9
Where more than one conductor is used per phase.
Capacitance between conductors : ENG348 Lect 11
Voltage between conductors : ENG348 Lect11 pg4

$$V_{ki} = \frac{1}{2\pi\,\varepsilon}\sum_{m=1}^{M}\rho_m\,\ell n\left(\frac{R_{im}}{R_{km}}\right)$$

Actual Equation : eng348 Lect11 pg5

$$V_{ki} = \frac{1}{2\pi\,\varepsilon}\sum_{m=1}^{M}\rho_m\,\ell n\left(\frac{D_{im}}{D_{km}}\right) \qquad (7)$$

Potential Difference between 2 conductors of same radius :
ENG348 Lect11 pg5

$$V_{12} = \frac{\rho}{\pi\,\varepsilon}\,\ell n\left(\frac{D}{r}\right) \quad V$$ where ρ = *Charge Density*

Capacitance per unit length : eng348 Lect11 pg6

$$C'_{12} = \frac{C_{12}}{l} = \frac{\pi\epsilon}{\ln\left(\frac{D}{r}\right)} \quad F/m$$

(11)

where	D	=	Distance between conductors
and	r	=	Radius of conductor

Surface Fields : **ENG348 Lect11 pg6**
Single phase based on equation (8)

$$E_{surface} = \frac{V_{12}}{2r\,\ell n\left(\frac{D}{r}\right)} \quad V/m \qquad (14)$$

3 phase based on equation (20)

$$E_{surface} = \frac{V_{an}}{r\,\ell n\left(\frac{D_{eq}}{r}\right)} \quad V/m \qquad (26)$$

Line to Neutral Capacitance : ENG348 Lect11 p9
The algebraic manipulation leading to this formula in the notes is fairly easy to follow.

$$C_{an}' = \frac{\rho}{V_{an}} = \frac{2\pi\,\varepsilon}{\ell n\left(\frac{D}{r}\right)} \quad F/m \qquad (21)$$

If the conductor spacing is not even then insert $GMD = \sqrt[3]{D_{ab}\,D_{bc}\,D_{ca}}$ into equation (21) above to create equation (23) (**ENG348 Lect11 p10**) below

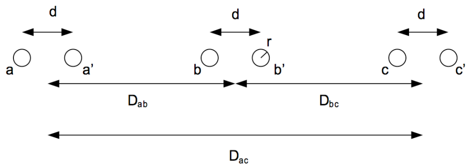
$$C_{an}' = \frac{2\pi\,\varepsilon}{\ell n\left(\frac{GMD}{r}\right)} \quad F/m \qquad (23)$$

If bundled conductors are used for each phase then each bundle can be represented by :

$$R_{SC} = \sqrt{r\,d}$$

(24)
and equation (23) (**ENG348 Lect11 p10**) becomes

$$C_{an}' = \frac{2\pi\,\varepsilon}{\ell n\left(\frac{GMD}{R_{SC}}\right)} \quad F/m \qquad (25)$$



Transmission Parameters ABCD : ENG348 lect12 pg 2

Line Impedance : ENG348 lect12 p2
Total line impedance : $Z = (R + j\omega L)l \ \ \Omega$ (4)
where l = the length of line in km

Line Admittance: ENG348 lect12 p3
For longer lengths, up to about 250 km, the effects of the shunt admittance become importance. If we define the shunt admittance as :

$$Y = (G + j\omega C)l \ \ S(siemens) \qquad (6)$$

or these values can be found from (**ENG348 lect12 p4**)

$$Z = B$$

$$Y = \gamma = \frac{2\times(A-1)}{Z}$$

(11)
Which fits into

$$\begin{bmatrix}\mathbf{V}_S\\ \mathbf{I}_S\end{bmatrix} = \begin{bmatrix}1 & Z\\ 0 & 1\end{bmatrix}\begin{bmatrix}\mathbf{V}_R\\ \mathbf{I}_R\end{bmatrix} \qquad (5)$$

This model is usually acceptable for overhead 60-Hz lines with line lengths of less that about 80 km.

Medium Length Model For a transmission line : **ENG348 lect12 p4**

$$\begin{bmatrix}\mathbf{V}_S\\ \mathbf{I}_S\end{bmatrix} = \begin{bmatrix}\left(1 + \frac{YZ}{2}\right) & Z\\ Y\left(1 + \frac{YZ}{4}\right) & \left(1 + \frac{YZ}{2}\right)\end{bmatrix}\begin{bmatrix}\mathbf{V}_R\\ \mathbf{I}_R\end{bmatrix} \qquad (10)$$

NOTE: Working in reverse, if the ABCD parameters are known, then the series and shunt elements can be found from:

$$Z = B$$

$$Y = \gamma = \frac{2\times(A-1)}{Z}$$

(11)

No Load Voltage for a transmission line : **ENG348 lect12 p5**

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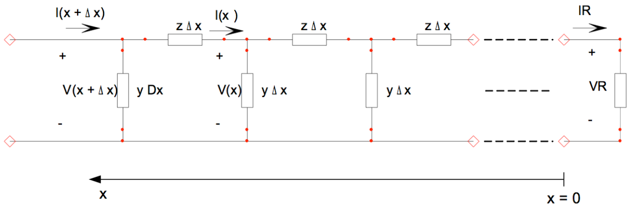
Using ABCD parameters, the no-load voltage is given by :

$$V_{BNL} = \frac{V_S}{A}$$

No Load Voltage for a transmission line : **ENG**348 lect12 p6

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \quad \text{(2) eng348 lect12 p1}$$

$$= \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$



How to find ABCD in a medium line : ENG348 lect12 p6

1. Find the **propagation constant** :

$$\gamma = \sqrt{z\,y} \quad m^{-1} \quad \text{(15)}$$

which will generally be a complex #

2. Find the **characteristic impedance** :

$$Z_c = \sqrt{\frac{z}{y}} \ \Omega \quad \text{(16)}$$

3. Find the **Hyperbolic Identities : ENG**348 lect12 p7

$$\cosh(\gamma\ x) = \frac{1}{2}\left(e^{\gamma\ x} + e^{-\gamma\ x}\right)$$

$$\sinh(\gamma\ x) = \frac{1}{2}\left(e^{\gamma\ x} - e^{-\gamma\ x}\right) \quad \text{(17)}$$

4. Then insert these values into :

$$\begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} \cosh(\gamma\ x) & Z_c \sinh(\gamma\ x) \\ \frac{1}{Z_c}\sinh(\gamma\ x) & \cosh(\gamma\ x) \end{bmatrix} \begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix} \quad \text{(14)}$$

Voltage Regulation – **ENG347 p6L12**

$$V_{s.no.load} = \frac{V_P}{\left(\frac{N_P}{N_S}\right)} \qquad VR = \frac{V_{s.no.load} - V_{s.full.load}}{V_{s.full.load}} \times 100$$

In Lossless lines : **ENG348 SG13 p4**

$$\mathbf{V}_S = A\mathbf{V}_{RNL} + B0$$

$$\therefore \ \mathbf{V}_{RNL} = \frac{\mathbf{V}_S}{A} = \frac{356.9\angle 16.12^o}{0.9295} = 384.0\angle 16.12^o$$

Lossless lines : **ENG**348 lect13 p1

β is in Radians/km.

$$\mathbf{V}(x) = \cos(\beta\ x)\mathbf{V}_R + jZ_c\sin(\beta\ x)\mathbf{I}_R$$

$$\mathbf{I}(x) = \frac{j\sin(\beta\ x)}{Z_c}\mathbf{V}_R + \cos(\beta\ x)\mathbf{I}_R$$

where

$$\gamma = j\beta = j2\pi\ f\sqrt{LC} \quad m^{-1}$$

and

$$Z_c = \sqrt{\frac{L}{C}} \quad \Omega$$

β is in Radians/km.

Z_C = Surge impedance

$$\lambda = \frac{2\pi}{\beta}$$

When deciding whether to use these equations it depends on the length of your transmission line relative to a wavelength. This wavelength comes from the speed of light. If your line is not that long then it can be lumped into a simplified model.

Speed of light in free space : **ENG348 lect13 p1**

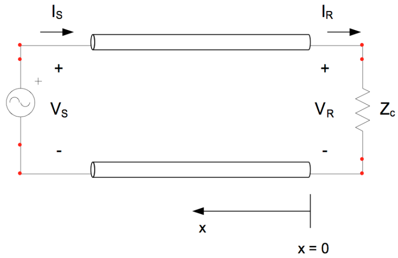
$$c = v = \frac{1}{\sqrt{(LC)}} = 3\times10^8 m/s$$

Lossless lines : ENG348 lect13 p2

$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cos(\beta\ l) & jZ_c\sin(\beta\ l) \\ \frac{j\sin(\beta\ l)}{Z_c} & \cos(\beta\ l) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix}$$

Surge Impedance Loading : ENG348 lect13 p2

Is the power delivered by a lossless line to a resistance equal to the surge impedance.



$$Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{z}{y}}\Omega$$

Because the voltage everywhere along the line are the same there is only a difference in phase shift. The phase shift is defined by the equations :

$$V_x = e^{j\beta x}V_R \quad (7)$$

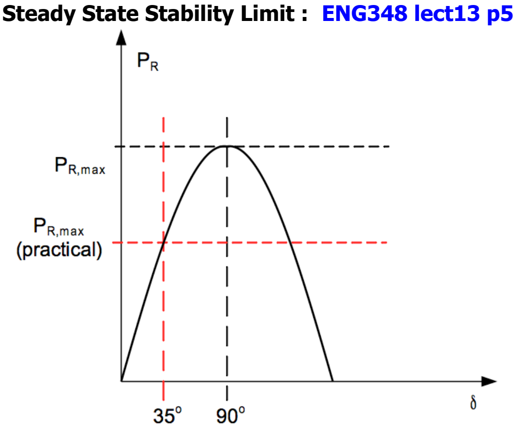
$$\mathbf{I}(x) = e^{j\beta\ x}\frac{\mathbf{V}_R}{Z_c} \quad amps \quad (8)$$

$$\mathbf{S}(x) = \mathbf{V}(x)\mathbf{I}^*(x)$$

$$= \frac{|\mathbf{V}_R|^2}{Z_c} \quad (9)$$

$$SIL \equiv P(0) = \frac{V_{rated}^2}{Z_c} \quad (10)$$

Steady State Stability Limit : ENG348 lect13 p5



$$P_R = \frac{V_S V_R \sin\delta}{Z_C \sin(\beta\ l)} \quad (14)$$

Finding ABCD in various line lengths :

How to find ABCD in a short line : ENG348 lect12 P??

How to find ABCD in a medium line : ENG348 lect12 p6

How to find ABCD in a lossless line : ENG348 lect13 p8

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$$\begin{bmatrix} V_S \\ I_S \end{bmatrix} = \begin{bmatrix} \cos(\beta\, l) & j Z_c \sin(\beta\, l) \\ \dfrac{j \sin(\beta\, l)}{Z_c} & \cos(\beta\, l) \end{bmatrix} \begin{bmatrix} V_R \\ I_R \end{bmatrix} \qquad (18)$$

where

$$\beta = 2\pi f \sqrt{LC} \quad m^{-1}$$

and β is in Radians/km.

$$Z_c = \sqrt{\frac{L}{C}} \quad \Omega \qquad\qquad Z_c = \sqrt{\frac{L}{C}} = \sqrt{\frac{z}{y}}\Omega$$

(19)

For a short line, $\beta\, l << \pi$, then

$$\cos(\beta\, l) \cong 1$$

and (20)

$$\sin(\beta\, l) \cong \beta\, l$$

ENG348 lect13 p9

Reactance (X) is the imaginary part of **Impedance** (Z)

Susceptance (B) is the imaginary part of **Admittance** (Y)

Trig identity needed : eng348 lect13 p12

$$\sin^2(-\delta) + \cos^2(-\delta) = \left(\frac{P_R X_L}{V_R}\right)^2 + \left(\frac{Q_R X_L}{V_R} + V_R\right)^2$$

- ***Shunt inductors***: these absorb reactive power and reduce overvoltages during light load conditions. They are usually removed under full load conditions.
- ***Shunt capacitors***: these deliver reactive power and increase voltages during heavy load conditions.
- ***Static var compensators***: these can automatically adjust the compensation to maintain voltage levels and loadability over a wide range of loading conditions.
- ***Series capacitors***: these reduce the net series impedance of the line, which reduces line-voltage drops and increases line loadability.

Analytic Power Flow equations : ENG348 lect 14 p5

$$P_2 = Y_{TL} V_2^2 \cos\theta_{TL} - Y_{TL} V_2 \cos(\theta_{TL} - \delta_2)$$

$$Q_2 = - Y_{TL} V_2^2 \sin\theta_{TL} + Y_{TL} V_2 \sin(\theta_{TL} - \delta_2)$$

(13a)

(13b)

Iterative Power Flow – solution system : ENG348 lect 14 p12

An iterative procedure to solve this problem is given below:

1. Provide an initial guess for the phase angle δ_2, say $\delta_2 = 0$
2. Compute the reactive power Q_{G2} using equation (24).
3. Update the bus 2 voltage using equation (23).
4. From step 3, the bus 2 voltage takes the form $\mathbf{V}_2 = V_2 \angle \delta$. Set $V_2 = 1$, so that the new value for the bus voltage becomes $\mathbf{V}_2 = 1 \angle \delta$.
5. If the changes in Q_{G2} and δ_2 are small, go to step (6), otherwise return to step 2.
6. Compute power delivered by generator 1:

$$S_1^* = P_1 - jQ_1 = \frac{1 - \mathbf{V}_2}{j0.5} \qquad (25)$$

Gauss-Seidel Approach : ENG348 lect15 p3

$$V_k = \frac{1}{Y_{kk}} \left(\frac{P_k - jQ_k}{V_k^*} - \sum_{\substack{n=1 \\ n \neq k}}^{N} Y_{kn} V_n \right) \qquad (8)$$

Combatting reactances : ENG348 lect16 p3

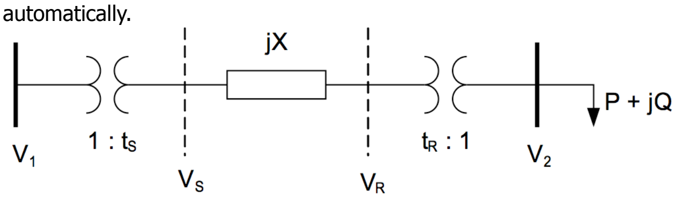
If the load reactive power is +ve then use a capacitor in **Parallel.**
If the load reactive power is -ve then use a capacitor in **Series.**
Problems with the series capacitance can be overcome with an **over-voltage** relay.

Synchronous Motors : ENG348 lect16 p3

In most plants induction motors are used creating an overall lagging power factor. Synchronous motors can be used to bring the plant to near unity power factor. The motor in these applications is sometimes called a **synchronous capacitor.**

Tap Changing Transformers : ENG348 Lect16 p7

Ability to change the turns ratio to allow for voltage drops along transmission lines. Tap Changing Under Load (TCUL) transformers do this automatically.



$$t_S = \sqrt{\frac{\dfrac{V_2}{V_1}}{1 - \dfrac{XQ}{V_1 V_2}}}$$

(17)

Fault Levels : ENG348 lect 18 p7

$$FL = \sqrt{(3)}\, V_{\text{nominal}}\, I_f$$

(4)

$V_{nominal}$ is the Voltage between 2 points and
I_f is the short circuit current on the nominal voltage.

Symmetrical Components : ENG348 Lect19 p3

$$\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_0 \\ \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} \qquad (9)$$

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

ENG348 Lect19 p4 : the first one is the base equation we use.
The current I_a I_b I_c are the line to neutral currents.

$$\mathbf{I}_p = \begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_0 \\ \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \mathbf{A}\,\mathbf{I}_s$$

and

$$\mathbf{I}_s = \begin{bmatrix} \mathbf{I}_0 \\ \mathbf{I}_1 \\ \mathbf{I}_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} = \mathbf{A}\,\mathbf{I}_p$$

Where $a = 1 \angle 120^o$ and

$a^2 = 1 \angle 240^o$

Identity for a : ENG348 lect19 p2

$$1 + a + a^2 = 0$$

Neutral Node Current. - If the neutral node is grounded then you can do KCL to find the neutral node current. ENG348 Lect 19 page 8

$$I_n = I_b + I_c$$

Gauss Seidel - ENG348 lect 15 page 1

An iterative method is needed because the problem is non-linear, with a higher order of complexity than a simple quadratic.

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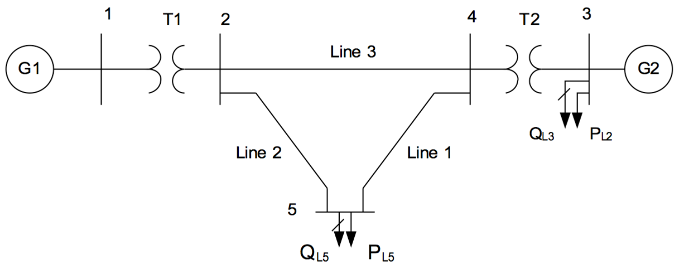
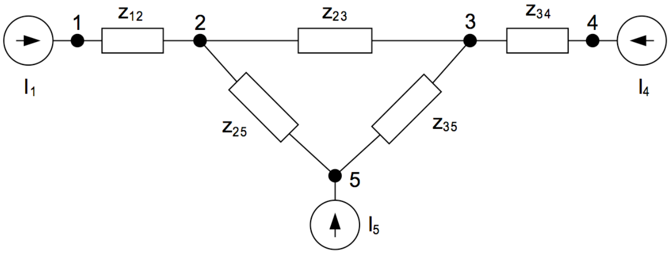


To analyse this circuit

- Convert the Sources and Loads into currents and
- the lines and transformers into impedances.



Then do KCL at each load but use the admittances instead of impedances.

Ie $y_{12} = \dfrac{1}{Z_{12}}$ to finally create a matrix like

$$\begin{bmatrix} I_1 \\ 0 \\ 0 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} y_{12} & -y_{12} & 0 & 0 & 0 \\ -y_{12} & y_{12}+y_{23}+y_{25} & -y_{23} & 0 & -y_{25} \\ 0 & -y_{23} & y_{23}+y_{34}+y_{35} & -y_{34} & -y_{35} \\ 0 & 0 & -y_{34} & y_{34} & 0 \\ 0 & -y_{25} & -y_{35} & 0 & y_{25}+y_{35} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \end{bmatrix}$$

To finally solve the matrix we assume a number for all the variables except one and solve for that. Then we insert the answer and re-iterate with another variable until convergence or divergence is reached.

NB: If the equations are diverging then invert them to see if they start to converge. (see ENG348 Lect 15 p6)

Analysis of Fault Currents – ENG348 lect 18 pages 4 to 7

Example [Glover P7.14]

Equipment ratings for the five-bus power system shown in figure 6 are as follows:

Generator G1:	50 MVA,	12kV,	$X'' = 0.2$ per unit
Generator G2:	100 MVA,	15 kV,	$X'' = 0.2$ per unit
Transformer T1:	50 MVA,	10 kV Y/ 138 kV Y,	$X = 0.10$ per unit
Transformer T2:	100 MVA,	15 kV Δ/ 138 kV Y,	$X = 0.10$ per unit
Each 138-kV line:	$X_l = 40\ \Omega$		

A three-phase short-circuit occurs at bus 5, where the pre-fault voltage is 15 kV. Pre-fault load current is neglected.

(a) Draw the positive-sequence reactance diagram in per-unit on a 100-MVA, 15-kV base in the zone of generator G2.

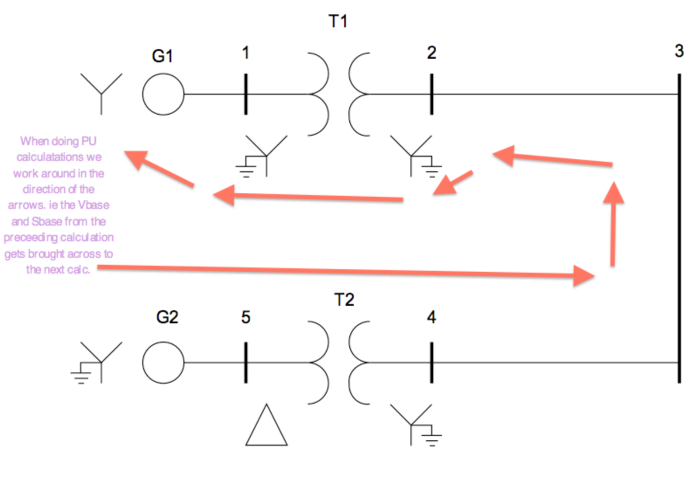


Figure 6

First, all rated impedances must be re-scaled for a common 100-MVA base, and a 15-kVA base in the zone of generator G2. This gives per-unit impedances of:

$$X''_{g2} = X'' = 0.2\ pu$$

$$X''_{t2} = X = 0.1\ pu$$

$$X_{tl2} = 40 \times \frac{100}{138^2} = 0.210\ pu$$

$$X_{tl1} = 40 \times \frac{100}{138^2} = 0.210\ pu$$

$$X''_{t1} = X'' \times \frac{S_{new}}{S_{old}} = 0.1 \times \frac{100}{50} = 0.2\ pu$$

$$X''_{g2} = X'' \times \frac{S_{new}}{V^2_{new}} \times \frac{V^2_{old}}{S_{old}} = 0.2 \times \frac{100}{10^2} \times \frac{12^2}{50} = 0.576\ pu$$

Where did these values come from?
100MVA is from above and the 50 is specified in the original specs in the beginning of the question. Sbase is used instead of Zbase because the Vbase is the same as the Vbase from the preceeding calculation.

g1

Xg2newpu = X * Zold/Znew

The positive-sequence reactance diagram is shown in figure 7.

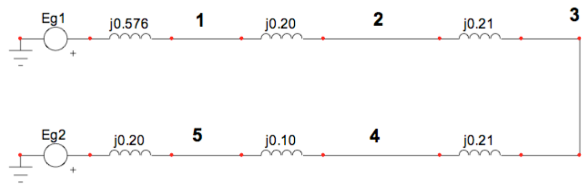


Figure 7

The short-circuit fault occurs at bus 5. The fault-circuit for this event is shown in figure 8, where X_r represents the sum of the reactances to the right of the fault.

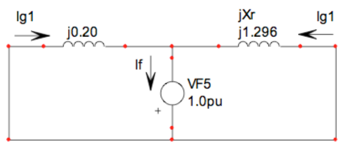


Figure 8

We could form the parallel combination of the two reactances into a single equivalent reactance, but it is just as easy to find I_{g1} and I_{g2} and then find the subtransient fault current I_f.

Are we just assuming the current is 1pu? This is voltage, not current.

$$I_{g1} = \frac{1}{j0.2} = -j5.0\ pu$$

$$I_{g2} = \frac{1}{j1.296} = -j0.7716\ pu$$

$$I_f = I_{g1} + I_{g2} = -j5.7716\ pu$$

The current base at bus 5 is

$$I_{base5} = \frac{S_{base}}{\sqrt{3}\,V_{base}} = \frac{100 \times 10^6}{\sqrt{3} \times 15 \times 10^3} = 3.849\ kA$$

$$\therefore\ I_f = -j5.7716 \times 3.849 = -j22.2\ kA$$

$$\therefore\ I_{g1} = -j0.7716 \times 3.849 = -j2.97\ kA$$

$$\therefore\ I_{g2} = -j5.0 \times 3.849 = -j19.2\ kA$$

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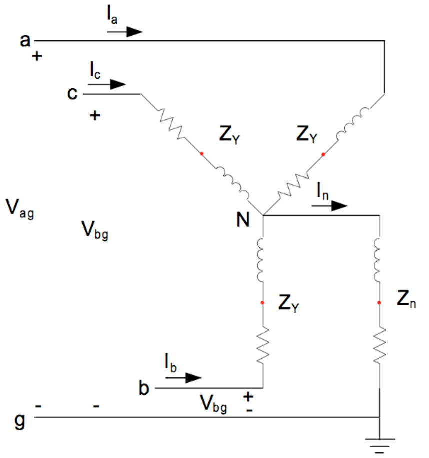
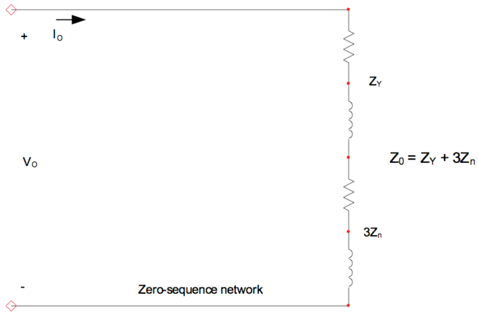
Sequence networks : ENG348 lect 20 page 1

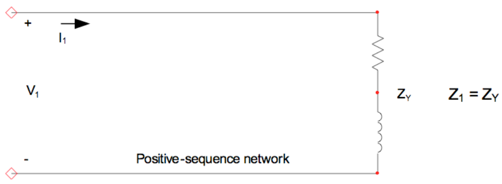
The equivalent conversion from the $V_{ag,bg,cg}$ normal network to the sequence network $V_{0,1,2}$ is as follows.

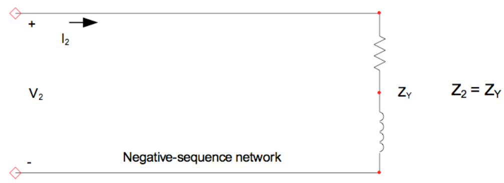


Conversions as follows



Note here that it's $3Z_n$ and not just Z_n. This is because it's the multiplication of 3 currents coming into that node.





ENG348 SG13 Q6

A three-phase, 60-Hz, 500-kV transmission line is 300 km long. The line's phase constant is $\beta = 0.00125$ rad/km and its surge impedance is $Z_c = 290\ \Omega$.

(a) Find the line inductance in mH/km per phase, and its capacitance in µF/km per phase.
(b) Find the *A*,*B*,*C*,*D* parameters for the given length of line.
(c) The receiving end rated load is 800 MW, 0.8 power factor lagging at 500 kV. Determine the sending end *phase* voltage.
(d) If the line is terminated in an open circuit and is energised with 500 kV at the sending end, then calculate

(i) the receiving end *phase* voltage;
(ii) the reactance and the Mvar of a three-phase shunt inductance that is installed at the receiving end to keep the no-load receiving end voltage at the rated value.

(a)
$$\beta = 2\pi \times 60\sqrt{LC}$$
$$\therefore LC = \left(\frac{0.00125}{2\pi \times 60}\right)^2 = 10.99 \times 10^{-12}$$
$$Z_c = \sqrt{\frac{L}{C}}$$
$$\therefore \frac{L}{C} = 290^2 = 84100$$
$$\therefore L = 84100 \times C$$
$$\therefore 84100 \times C^2 = 10.99 \times 10^{-12}$$
$$\therefore C = 0.0114\ \mu F/km$$
$$\therefore L = 0.961\ mH/km$$

(b)
$$A = D = \cos(\beta l) = \cos(0.00125 \times 300) = 0.931$$
$$B = jZ_c \sin(\beta l) = j290 \times \sin(0.00125 \times 300) = j106.2$$
$$C = j\frac{1}{Z_c}\sin(\beta l) = j\frac{\sin(0.00125 \times 300)}{290} = j0.001263$$

(c)
$$V_R = \frac{500 \times 10^3}{\sqrt{3}} \angle 0^\circ = 288.7 \angle 0^\circ\ kV$$

Why is this 0.8 here. I know it's the power factor, but why is it being used.?

$$I_R = \frac{800 \times 10^6}{3 \times 288.7 \times 10^3 \times 0.8} \angle -\cos^{-1}(0.8) = 1155 \angle -36.87^\circ\ A$$
$$V_S = 0.931 \times 288.7 \times 10^3 \angle 0^\circ + (106.2\angle 90^\circ) \times (1155 \angle -36.87^\circ)$$
$$= 268.78 + 73.60 + j98.13$$
$$= 356.2 \angle 16.0^\circ\ kV$$

Gregs Email says :
For this question, the load data is given as three-phase power, power factor, and line-to-line voltage.

Now, P = sqrt(3)xVxIxPF
Therefore I = P/(sqrt(3)xVxPF)

(d)
$$I_R = 0$$
$$\therefore V_S = A\, V_R$$
$$\therefore V_R = \frac{V_S}{A} = \frac{500\angle 0^\circ}{\sqrt{3} \times 0.931} = 310.1\angle 0^\circ\ kV$$

(e)
$$V_R = 288.7\angle\delta$$
$$I_R = \frac{288.7\angle\delta}{X\angle 90^\circ}$$
$$\therefore 288.7 = 0.931 \times 288.7\angle\delta + (106.2\angle 90^\circ)\left(\frac{288.7}{X}\angle(\delta - 90^\circ)\right)$$

Require $\delta = 0$

and $288.7 = 268.8 + \dfrac{106.2 \times 288.7}{X}$

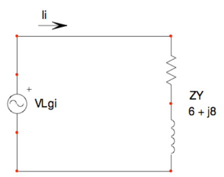
$$\therefore X = 1541\ \Omega$$

ENG348 SG20 Q1 for the exam

[Glover P8.14]
The line-to-ground voltages $V_{ag} = 280\angle 0^\circ$, $V_{bg} = 290\angle -130^\circ$, and $V_{cg} = 260\angle 110^\circ$ volts are applied to a balanced-Y load consisting of $(6 + j8)$ ohms per phase. The load neutral is solidly grounded. Draw the sequence networks and calculate $\mathbf{I}_0$, $\mathbf{I}_1$, and $\mathbf{I}_2$, the sequence components of the line currents. Then calculate the line currents $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$.

[*NOTE: the sequence voltages corresponding to the given line-to-ground voltages are* $V_{Lg0} = 7.530\angle 78.07^\circ$, $V_{Lg1} = 275.7\angle -6.63^\circ$, *and* $V_{Lg2} = 24.85\angle 79.40^\circ$ *volts*]

(a) The neutral is solidly grounded, so that $Z_n = 0$. All sequence networks have the same form shown below.



$$Z_Y = 6 + j8 = 10\angle 53.13^\circ\ \Omega$$
$$\mathbf{I}_0 = \frac{\mathbf{V}_{Lg0}}{Z_Y} = \frac{7.53\angle 78.07^\circ}{10\angle 53.13^\circ} = 0.753\angle 24.94^\circ\ A$$
$$\mathbf{I}_1 = \frac{\mathbf{V}_{Lg1}}{Z_Y} = \frac{275.7\angle -6.63^\circ}{10\angle 53.13^\circ} = 27.57\angle -59.76^\circ\ A$$
$$\mathbf{I}_2 = \frac{\mathbf{V}_{Lg2}}{Z_Y} = \frac{24.85\angle 79.40^\circ}{10\angle 53.13^\circ} = 2.485\angle 26.27^\circ\ A$$

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$$\begin{aligned}
\mathbf{I}_a &= \mathbf{I}_0 + \mathbf{I}_1 + \mathbf{I}_2 \\
&= 0.763\angle 24.94^\circ + 27.57\angle -59.76^\circ + 2.485\angle 26.27^\circ \\
&= (0.6919 + j0.3217) + (13.89 - j23.82) + (2.228 + j1.100) \\
&= 16.81 - j22.40 \\
&= 28.01\angle -53.11^\circ \quad A
\end{aligned}$$

$$\begin{aligned}
\mathbf{I}_b &= \mathbf{I}_0 + a^2\mathbf{I}_1 + a\mathbf{I}_2 \\
&= 0.763\angle 24.94^\circ + (1\angle +240^\circ)\times 27.57\angle -59.76^\circ + (1\angle +120^\circ)\times 2.485\angle 26.27^\circ \\
&= (0.6919 + j0.3217) + (-27.6 - j0.115) + (-2.067 + j1.380) \\
&= -29.0 - j1.59 \\
&= 29.0\angle -176.9^\circ \quad A
\end{aligned}$$

$$\begin{aligned}
\mathbf{I}_c &= \mathbf{I}_0 + a\mathbf{I}_1 + a^2\mathbf{I}_2 \\
&= 0.763\angle 24.94^\circ + (1\angle +120^\circ)\times 27.57\angle -59.76^\circ + (1\angle +240^\circ)\times 2.485\angle 26.27^\circ \\
&= (0.6919 + j0.3217) + (13.69 + j23.93) + (-0.162 - j2.48) \\
&= 14.22 + j21.77 \\
&= 26.0\angle +58.85^\circ \quad A
\end{aligned}$$

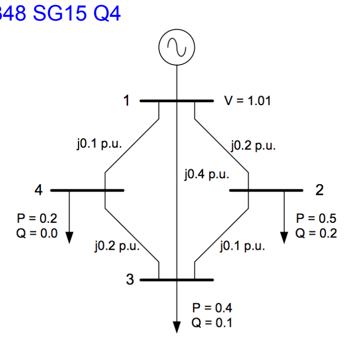
NOTE: In this case, with the neutrals connected together, the line currents can be found from the individual one-phase circuits. For example, $\mathbf{I}_b$ is given by

$$\mathbf{I}_b = \frac{\mathbf{V}_{bg}}{Z_Y} = \frac{290\angle -130^\circ}{10\angle 53.13} = 29.0\angle -183.13^\circ \quad A$$

KCL and Complex Power - ENG348 SG15 Q4

As all currents flowing into a point = 0 in KCL, but all complex powers in a circuit add to 0. Or in this case all power flowing into a node must equal 0.
At bus 2 in the following diagram we can write :

$$0 = S_{12} - S_{out} + S_{32}$$



(a) An iterative method is needed because the problem is non-linear, with a higher order of complexity than a simple quadratic.
(b) KCL at bus 1 is

$$\begin{aligned}
\mathbf{I}_1 &= \frac{\mathbf{V}_1 - \mathbf{V}_4}{j0.1} + \frac{\mathbf{V}_1 - \mathbf{V}_3}{j0.4} + \frac{\mathbf{V}_1 - \mathbf{V}_2}{j0.2} \\
&= (-j10 - j2.5 - j5)\,\mathbf{V}_1 - (-j10)\mathbf{V}_4 - (-j2.5)\mathbf{V}_3 - (-j5)\mathbf{V}_2 \\
&= -j17.5\,\mathbf{V}_1 + j5\,\mathbf{V}_2 + j2.5\,\mathbf{V}_3 + j10\,\mathbf{V}_4
\end{aligned}$$

where $\mathbf{I}_1$ is the current entering the bus from the generator. This equation could be obtained by inspection. Applying inspection to the remaining buses, the bus admittance matrix becomes:

$$\begin{bmatrix}\mathbf{I}_1\\ \mathbf{I}_2\\ \mathbf{I}_3\\ \mathbf{I}_4\end{bmatrix} = \begin{bmatrix} -j17.5 & j5 & j2.5 & j10 \\ j5 & -j15 & j10 & 0 \\ j2.5 & j10 & -j17.5 & j5 \\ j10 & 0 & j5 & -j15 \end{bmatrix}\begin{bmatrix}\mathbf{V}_1\\ \mathbf{V}_2\\ \mathbf{V}_3\\ \mathbf{V}_4\end{bmatrix}$$

(c) At bus 2:

$$S_{in} = S_o + S_{32}$$

$$V_2\left(\frac{V_1 - V_2}{j0.2}\right)^* = V_2\left(\frac{V_2 - V_3}{j0.1}\right)^* + (0.5 + j\,0.2)$$

$$\therefore\ V_2^*\left(\frac{V_1 - V_2}{j0.2}\right) = V_2^*\left(\frac{V_2 - V_3}{j0.1}\right) + (0.5 - j\,0.2)$$

$$\therefore\ V_2^*\left(\frac{V_1 - V_2}{j0.2} + \frac{V_3 - V_2}{j0.1}\right) = (0.5 - j\,0.2)$$

$$\therefore\ j15\,V_2 - j5\,V_1 - j10\,V_3 = \frac{(0.5 - j\,0.2)}{V_2^*}$$

$$\therefore\ V_2 = \frac{1}{j15}\left(\frac{(0.5 - j\,0.2)}{V_2^*} + j5\,V_1 + j10\,V_3\right) \quad \ldots(1)$$

$$V_3 = \frac{1}{j17.5}\left(\frac{(0.4 - j\,0.1)}{V_3^*} + j2.5\,V_1 + j10\,V_2 + j5\,V_4\right) \quad (2)$$

$$V_4 = \frac{1}{j15}\left(\frac{0.2}{V_4^*} + j10\,V_1 + j5\,V_3\right) \quad \ldots(3)$$

The voltage for the slack bus is $\mathbf{V}_1 = 1.01\angle 0^\circ\ p.u.$

To begin, assume that $\mathbf{V}_2 = \mathbf{V}_3 = \mathbf{V}_4 = 1.01\angle 0^\circ\ p.u.$

Start the iterations by updating the value for $\mathbf{V}_2$: From eq.(1):

$$\begin{aligned}
\mathbf{V}_2 &= \frac{1}{j15}\left(\frac{(0.5 - j\,0.2)}{1.01} + j5\times 1.01 + j10\times 1.01\right) \\
&= 0.9968 - j0.033 \\
&= 0.9973\angle -1.896^\circ \quad p.u.
\end{aligned}$$

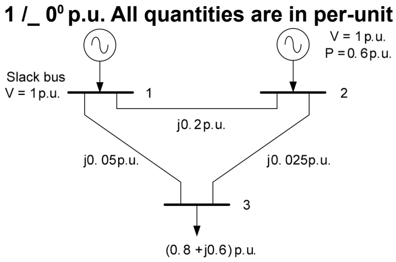
From eq.(2):

$$\begin{aligned}
\mathbf{V}_3 &= \frac{1}{j17.5}\left(\frac{(0.4 - j\,0.1)}{1.01} + j2.5\times 1.01 + j10\times(0.9968 - j0.033) + j5\times 1.01\right) \\
&= 0.9968 - j\,0.04149 \\
&= 0.9977\angle -2.384^\circ \quad p.u.
\end{aligned}$$

$$\begin{aligned}
\mathbf{V}_4 &= \frac{1}{j15}\left(\frac{0.2}{1.01} + j10\times(0.9968 - j0.033) + j5\times(0.9968 - j\,0.04149)\right) \\
&= 0.9968 - j\,0.04903 \\
&= 0.9980\angle -2.816^\circ \quad p.u.
\end{aligned}$$

ENG348 SG15 Q5

[Weedy & Cory, P6.7] Determine the voltage at bus 2 and the reactive power at bus 3 as shown in the figure below, after the first iteration of a Gauss-Seidel load flow method. Assume the initial voltage to be 1 /_ 0^0 p.u. All quantities are in per-unit on a common base.



At bus 3:

$$V_3 I_{13}^* + V_3 I_{23}^* = 0.8 + j0.6$$

$$\therefore\ V_3^*\left(\frac{V_1 - V_3}{j0.05} + \frac{V_2 - V_3}{j0.025}\right) = 0.8 - j0.6$$

$$\therefore\ V_3^*\left(-j20\,V_1 - j40\,V_2 + j60\,V_3\right) = 0.8 - j0.6$$

$$\therefore\ V_3 = \frac{1}{j60}\left(\frac{0.8 - j0.6}{V_3^*} + j20\,V_1 + j40\,V_2\right)$$

At bus 2:

$$0.6 + jQ = V_2 I_{21}^* + V_2 I_{23}^*$$

$$\therefore\ 0.6 - jQ = V_2^*\left(\frac{V_2 - V_1}{j0.2} + \frac{V_2 - V_3}{j0.025}\right)$$

$$\therefore\ 0.6 - jQ = V_2^*\left(j5.0\,V_1 - j45.0\,V_2 + j40\,V_3\right)$$

$$\therefore\ V_2 = -\frac{1}{j45}\left(\frac{0.6 - jQ}{V_2^*} - j5.0\,V_1 - j40\,V_3\right)$$

$$\therefore\ V_2 = \frac{1}{j45}\left(\frac{-0.6 + jQ}{V_2^*} + j5.0\,V_1 + j40\,V_3\right)$$

Also, because we will need to do a reactive power calculation:
From

$$0.6 - jQ = V_2^*\left(j5.0\,V_1 - j45.0\,V_2 + j40\,V_3\right)$$

$$\therefore\ Q = \mathrm{Im}\left[0.6 - V_2^*\left(j5.0\,V_1 - j45.0\,V_2 + j40\,V_3\right)\right]$$

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Using $\mathbf{V}_1 = 1.0$, the Gauss-Seidel process is to solve the follwoing sequence of equations repeatedly until convergence is obtained.

$$Q = \operatorname{Im}\left[0.6 - V_2^*\left(j5.0 - j45.0V_2 + j40V_3\right)\right]$$

$$V_2 = \frac{1}{j45}\left(\frac{-0.6 + jQ}{V_2^*} + j5.0 + j40V_3\right)$$

$$V_3 = \frac{1}{j60}\left(\frac{0.8 - j0.6}{V_3^*} + j20 + j40V_2\right)$$

From the equation for $\mathbf{V}_2$, we reset $|\mathbf{V}_2| = 1$, but keep the phase angle.

Starting with $V_2 = V_3 = 1.0\angle 0°$, the first iteration gives:

$$Q = \operatorname{Im}\left[0.6 - 1.0\left(j5.0 - j45.0 \times 1.0 + j40 \times 1.0\right)\right]$$
$$= 0$$

$$V_2 = \frac{1}{j45}\left(\frac{-0.6}{1.0} + j5.0 + j40 \times 1.0\right)$$
$$= 1 + j0.01333$$

$$V_3 = \frac{1}{j60}\left(\frac{0.8 - j0.6}{1.0} + j20 + j40 \times \left(1.0 + j0.01333\right)\right)$$
$$= 0.99 - j0.00445$$

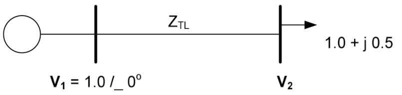
ENG348 SG3 Q3

In the two-bus system shown in the figure below, bus 1 voltage is $\mathbf{V}_1 = 1.0\angle 0°\ p.u.$ A load power of 1 + j0.5 p.u. is taken from bus 2. The line impedance is Z_{TL} = j0.16 *p.u.* When the power angle is small, the relationships between real and reactive powers and the bus voltages are given approximately by

$$P = \frac{V_1 V_2}{X}\sin\delta$$

$$V_1 - V_2 = \frac{XQ}{V_2}$$

Using these approximate expressions, determine the magnitude (in per unit) and phase of the voltage at bus 2, $\mathbf{V}_2$.



The second equation has only one unknown, which is V_2, so we solve that equation first:

$$1 - V_2 = \frac{0.16 \times 0.5}{V_2}$$

Rearranging:

$$V_2^2 - V_2 + 0.08 = 0$$

$$\therefore\ V_2 = \frac{1 \pm \sqrt{1 - 4 \times 0.08}}{2}$$

$$\therefore\ V_2 = 0.9123 \quad or \quad 0.0877$$

The larger value is used, so that $V_2 = 0.9123\,pu$. (The smaller voltage could occur in an event called *voltage collapse*.)

Now to find δ.

$$\sin\delta = \frac{PX}{V_1 V_2} = \frac{1.0 \times 0.16}{1.0 \times 0.9123} = 0.1754$$

$$\therefore\ \delta = \sin^{-1}0.1754 = 10.1°$$

In the derivation of the approximations, δ represents the angle by which $\mathbf{V}_L$ lags $\mathbf{V}_G$, so if $\mathbf{V}_G$ is the reference voltage then

$$\mathbf{V}_2 = 0.9123\angle -10.1° \quad pu$$

ENG348 SG3 Q4

A simple two-bus system is shown in the figure below, where the transmisison line is modelled by a pure reactance X. The load voltage phasor is taken as the reference phasor, with $\mathbf{V}_L = V_L\angle 0°$, while the generator voltage phasor is given by $\mathbf{V}_G = V_G\angle\delta$.

Shown that the load active power and reactive power are given by:

$$P = \frac{V_G V_L}{X}\sin\delta$$

$$Q = \frac{V_G V_L\cos\delta - V_L^2}{X}$$

Start with:

$$\mathbf{V}_L\mathbf{I}_L^* = P + jQ$$

where

$$\mathbf{I}_L = \frac{\mathbf{V}_G - \mathbf{V}_L}{jX} = \frac{\left(V_G\cos\delta - V_L\right) + jV_G\sin\delta}{jX}$$

ENG348 SG9 Q6

The per-phase line loss in a 40-km long transmission line is not to exceed 60kW while it is delivering 100A per phase. If the resistivity of the conductor material is 1.72 x 10^{-8} Ω.m, determine the required conductor diameter.

$$P_{loss} = I^2R = 100^2R = 60000$$

$$\therefore\ R = \frac{60000}{100^2} = 6\,\Omega$$

Now

$$R = \frac{\rho l}{A} = \frac{1.72 \times 10^{-8} \times 40 \times 10^3}{\pi \times \left(\frac{d}{2}\right)^2} = \frac{0.876 \times 10^{-3}}{d^2}$$

$$\therefore\ \frac{0.876 \times 10^{-3}}{d^2} = 6$$

$$\therefore\ d = \sqrt{\frac{0.876 \times 10^{-3}}{6}} = 12.08\,mm$$

ENG348 SG7 Q3

[Saadat, P3.3] A 24,000-kVA, 17.32-kV, 60-Hz, three-phase synchronous generator has a synchronous reactance of 5 Ω/phase and negligible armature resistance.

(a) At a certain excitation, the generator delivers rated load, 0.8 power factor lagging to the infinite bus bar at a line-to-line voltage of 17.32 kV. Determine the excitation voltage per phase.

(b) The excitation voltage is maintained at 13.4 kV/phase and the terminal voltage at 10 kV/phase. What is the maximum three-phase real power that the generator can develop before pulling out of synchronism?

(c) Determine the armature current for the condition of part (b).

(a)

Terminal voltage per phase $V_\phi = \frac{69.3}{\sqrt{3}} = 40.0\,kV$

Apparent power per phase: $S_\phi = \frac{60}{3} = 20MVA$

$$\therefore\ V_\phi I_\phi = 20 \times 10^6$$

$$\therefore\ I_\phi = \frac{20 \times 10^6}{40.0 \times 10^3} = 500\,A$$

If we choose the terminal phase volatge as the reference phasor then

$$\mathbf{V}_\phi = 40.0\angle 0° \quad kV$$

$$\mathbf{I}_\phi = 500\angle -\cos^{-1}(0.8) = 500\angle -36.9° \quad A$$

Generated emf:

$$\mathbf{E}_A = jX_S\mathbf{I}_\phi + \mathbf{V}_\phi$$
$$= (15\angle 90°) \times (500\angle -36.9°) + 40.0 \times 10^3$$
$$= 7.5 \times 10^3\angle 53.1° + 40.0 \times 10^3$$
$$= 4.50 \times 10^3 + j\,6.0 \times 10^3 + 40.0 \times 10^3$$
$$= 44.5 \times 10^3 + j\,6.0 \times 10^3$$
$$= 44.9\angle 7.68°\,kV$$

Hence the magnitude of the generated emf is 44.9 kV and the power angle is 7.68°.

(b)

Power delivered per phase is given by: $P_g = \frac{E_A V_\phi}{X_S}\sin\delta$

Maximum power occurs when $\delta = 0$: $P_g(\max) = \frac{36 \times 10^3 \times 40 \times 10^3}{15} \times 1 = 96\,MW$

$$P_{g,3\phi}(\max) = 3 \times 96 = 288\,MW$$

(c)

$$P_g = \frac{E_A V_\phi}{X_S}\sin\delta$$

$$\therefore\ \frac{48 \times 10^6}{3} = \frac{46 \times 10^3 \times 40 \times 10^3}{15} \times \sin\delta$$

$$\therefore\ \sin\delta = 0.130$$

$$\therefore\ \delta = 7.49°$$

Now

$$\mathbf{I}_\phi = \frac{\mathbf{E}_A - \mathbf{V}_\phi}{jX_S} = \frac{46\angle 7.49° - 40.0}{15\angle 90°}$$
$$= \frac{45.6 + j6.0 - 40.0}{15\angle 90°} = \frac{5.6 + j6.0}{15\angle 90°} = \frac{8.21\angle 47°}{15\angle 90°}$$
$$= 0.547\angle -43°\,kA$$

Power Factor = $\cos 43° = 0.731$ *lagging*

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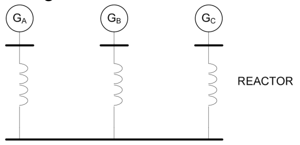
ENG348 SG22 Q3

[Saadat, P10.11] Three 15-MVA, 30-kV synchronous generators A, B, and C are connected via three reactors to a common bus bar, as shown in the figure below. The neutrals of generators A and B are solidly grounded, and the neutral of generator C is grounded through a reactor of 2.0 Ω. The generator data and the reactance of the reactors are tabulated below. A line-to-ground fault occurs on phase a of the common bus bar. Neglect pre-fault currents and assume generators are operating at their rated voltage. Determine the fault current in phase a.



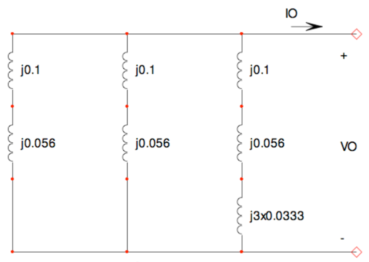
Item	X_1	X_2	X_0
G_A	0.25 pu	0.155 pu	0.056 pu
G_B	0.20 pu	0.155 pu	0.056 pu
G_C	0.20 pu	0.155 pu	0.060 pu
Reactor	6.0 Ω	6.0 Ω	6.0 Ω

ANS :

Reactor per unit reactances:

$$X = 6 \times \frac{15}{30^2} = 0.1\,pu$$

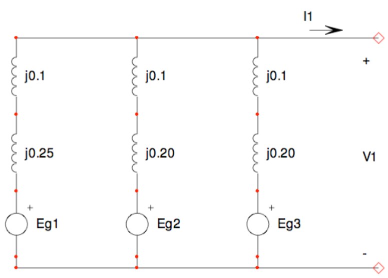
Sequence circuits (using the common bus as the output bus):



zero-phase sequence

Combining reactances:

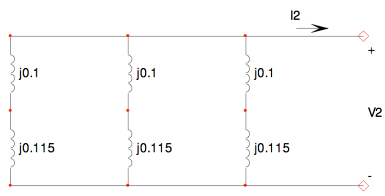
$$0.156||0.156||0.156 = 0.156||0.078 = \frac{0.156 \times 0.078}{0.156 + 0.078} = 0.059784\,pu$$



positive-phase sequence

Thevenin equivalent reactance:

$$0.35||0.30||0.30 = 0.35||0.15 = \frac{0.35 \times 0.15}{0.35 + 0.15} = 0.105\,pu$$

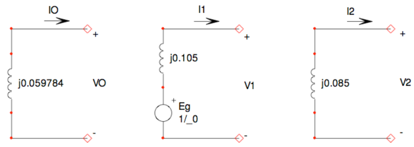


negative-phase sequence

Equivalent reactance:

$$0.255||0.255||0.255 = 0.255||0.1275 = \frac{0.255 \times 0.1275}{0.255 + 0.1275} = 0.085\,pu$$

The three simplified sequence circuits are:



Note: The Thevenin equivalent voltage for the positive sequence circuit is simply the rated generator voltage - this follows from the assumption to ignore prefault currents.

The output currents for phases *b* and *c* are zero. Under these conditions, the sequence currents are given by

$$I_0 = I_1 = I_2 = \frac{1}{3} I_a = \frac{1}{3} I_f$$

Hence the sequence voltages are given by:

$$V_0 = -j0.059784\, I_0 = -j0.059784\, \frac{I_f}{3}$$

$$V_1 = 1 - j0.105\, I_1 = 1 - j0.105\, \frac{I_f}{3}$$

$$V_2 = -j0.085\, I_2 = -j0.085\, \frac{I_f}{3}$$

As $V_a = 0$, and as V_a is equal to the sum of the sequence voltages, then

$$V_0 + V_1 + V_2 = \left(-j0.059784\,\frac{I_f}{3}\right) + \left(1 - j0.105\,\frac{I_f}{3}\right) + \left(-j0.085\,\frac{I_f}{3}\right) = 0$$

$$\therefore \left(j0.059784\,\frac{1}{3} + j0.105\,\frac{1}{3} + j0.085\,\frac{1}{3}\right) I_f = 1$$

$$\therefore I_f = \frac{1}{j0.083261} = -j12.0\ pu$$

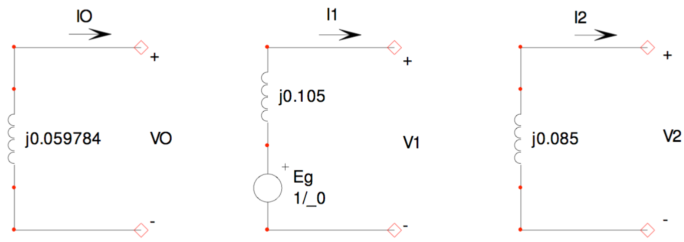
$$I_{base} = \frac{15}{\sqrt{3} \times 30} = 0.2887\,kA$$

$$\therefore I_f = 12 \times 0.2887 = 3.464\,kA$$

ENG348 SG22 Q4

[Saadat, P10.12] Three 15-MVA, 30-kV synchronous generators A, B, and C are connected via three reactors to a common bus bar, as shown in the figure below. The neutrals of generators A and B are solidly grounded, and the neutral of generator C is grounded through a reactor of 2.0 Ω. The generator data and the reactance of the reactors are tabulated below. A bolted line-to-line fault occurs between phases *b* and *c* on the common bus bar. Neglect prefault currents and assume generators are operating at their rated voltage. Determine the fault current that flows from phase *b* into phase *c*.

ANS : The sequence networks are given by :



The phase currents are given by

$$I_a = 0$$

$$I_b = -I_c = I_f$$

The sequence currents are given by

$$\begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ I_f \\ -I_f \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 0 \\ a\,I_f - a^2 I_f \\ a^2 I_f - a\,I_f \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 0 \\ j1.732 \\ -j1.732 \end{bmatrix} \times I_f$$

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From the sequence circuits:

$$V_0 = 0$$

$$V_1 = 1 - j0.105 \times \left(j1.732 \times \frac{I_f}{3} \right) = 1 + 0.18186 \frac{I_f}{3}$$

$$V_2 = -\left(j0.085\right) \times \left(-j1.732 \frac{I_f}{3} \right) = -0.14722 \frac{I_f}{3}$$

The phase voltages are given by

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ V_1 \\ V_2 \end{bmatrix}$$

As $V_b = V_c$ then

$$a^2 V_1 + a V_2 = a V_1 + a^2 V_2$$

$$\therefore \left(a^2 - a\right) V_1 = \left(a^2 - a\right) V_2$$

$$\therefore V_1 = V_2$$

$$\therefore 1 + 0.18186 \frac{I_f}{3} = -0.14722 \frac{I_f}{3}$$

$$\therefore 1 = -0.10969 \, I_f$$

$$\therefore I_f = \frac{1}{-0.10969} = -9.116 \, pu$$

$$\therefore I_f = -9.116 \times 0.2887 = 2.632 \, kA$$

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