

**References**

ENG125/243 Txtbk → Electrical Engineering – Fourth Ed.  
By Allan R. Hambley  
Other reference → Alexander, Fundamentals of Electric Circuits, 3rd Edition. McGraw Hill, 2007.  
Physics PEC152 → College Physics - 8<sup>th</sup> Ed By Vuille Serway  
Calculator used → Casio fx – 9860G AU  
Calculator book → Mathematics with a graphics calculator - Casio fx  
– 9860G Auby Barry Kissane & Marian Kemp

**Energy**

Watt = Joules/second = (Joules/Coulomb) x (Coulomb/second)  
Amp = (Coulomb/second)  
Volts = (joules/Coulomb)  
Coulomb = -1.602 x 10<sup>-19</sup> C

**Power** – P35 PEC152 55.3  $P = i^2 R$   
55.4  $P = v^2 / R$   
55.5  $P = Vi$

**Average Power through a resistance based on voltage** hamb pg 202

$$P_{avg} = \frac{V_{rms}^2}{R}$$

**Average Power through a resistance based on current** pg 202

$$P_{avg} = I_{rms}^2 R$$

**Average power Calculation** hamb pg228

$$P = V_{rms} I_{rms} \cos(\theta) \text{ or } P_{avg} = I_{rms}^2 R \text{ pg 202}$$

**P35 Calculation of Power**

$$P = i^2 R \quad P = \frac{v^2}{R} \quad P = Vi$$

$$S = \frac{1}{2} VI^* \text{ Page 23 - r2010-04-Power_Analysis.pdf}$$

All components in a circuit will add to zero. **This is whether they are Series or Parallel.**

$$P_1 + P_2 + P_3 = 0$$

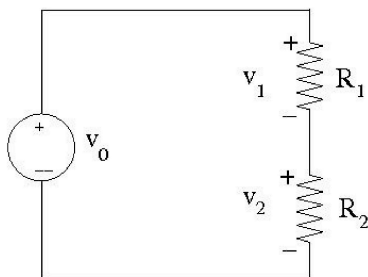
**Numbers**

Gig	<b>G</b>	10 <sup>9</sup>
Mega	<b>M</b>	10 <sup>6</sup>
Kilo	<b>k</b>	10 <sup>3</sup>
Milli	<b>m</b>	10 <sup>-3</sup>
Micro	<b>μ</b>	10 <sup>-6</sup>
Nano	<b>n</b>	10 <sup>-9</sup>
Pico	<b>p</b>	10 <sup>-12</sup>
Femto	<b>f</b>	10 <sup>-15</sup>

**Potential Divider**

$$v_1 = \frac{R_1}{R_1 + R_2} v_0$$

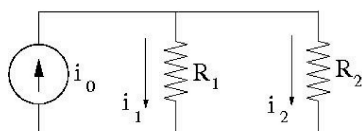
$$v_2 = \frac{R_2}{R_1 + R_2} v_0$$



**Current Divider**

$$i_1 = \frac{R_2}{R_1 + R_2} i_0$$

$$i_2 = \frac{R_1}{R_1 + R_2} i_0$$



**KCL pg 19**

**Kirchoff's Current Law**

The sum of all currents entering a node must equal the sum of all currents exiting the node. - **Add nodal drawing off printed sheet**

**Electric Current** Pg 570 Serwqy  $I_{av} = \frac{\Delta Q}{\Delta T}$

**KVL pg 22**

**Kirchoff's Voltage Law**

The sum of all currents in a closed loop will equal zero.

**Charge on a capacitor** 54.05  $q = Cv$  page122

**Capacitance of plate capacitor** 54.1  $C = \frac{\epsilon A}{d}$  page131

Murdoch University – ENG125 – Gareth Lee

Relation	Resistor	Capacitor (C)	Inductor
v-i:	(R) $v = iR$	$v = \frac{1}{C} \int i dt + v(t_0)$	(L) $v = L \frac{di}{dt}$
i-v:	$i = \frac{v}{R}$	$i = C \frac{dv}{dt}$	$i = \frac{1}{L} \int v dt + i(t_0)$
p or w:	$p = i^2 R = \frac{v^2}{R}$	$w = \frac{1}{2} Cv^2$	$w = \frac{1}{2} Li^2$
Series:	$R_{eq} = R_1 + R_2$	$C_{eq} = \frac{C_1 C_2}{C_1 + C_2}$	$L_{eq} = L_1 + L_2$
Parallel:	$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$	$C_{eq} = C_1 + C_2$	$L_{eq} = \frac{L_1 L_2}{L_1 + L_2}$
At dc:	Same	Open circuit	Short
Circuit var. that cannot change	n/a	v	$\phi$ circuit <sup>42</sup>

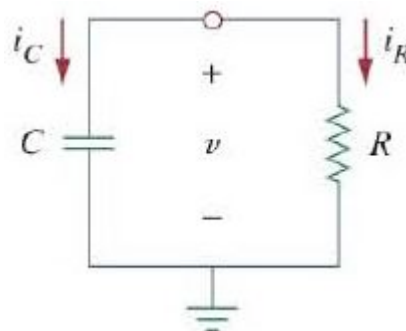
**Parallel Rule :**

$$\frac{1}{\frac{1}{2} + \frac{1}{3}} = \frac{2 \times 3 \times 1}{3 \times 2 + 2 \times 3} = \frac{2 \times 3}{6 + 6} = \frac{2 \times 3}{3 + 3}$$

When working out the capacitance or inductance in a circuit, try to stay away from the integration calculations to reduce the amount of steps needed to solve the circuit.

**Capacitance RC Circuits**

For capacitance circuits use KCL. Use the above table to substitute in for the currents.



1. Figure out Tau  $\tau = RC$

2. Everything is based on variants of this equation  $V = Ke^{-t/\tau}$

K = V<sub>(0)</sub> = the initial voltage

$\tau$  = Tau is the time constant

t = is the point in time

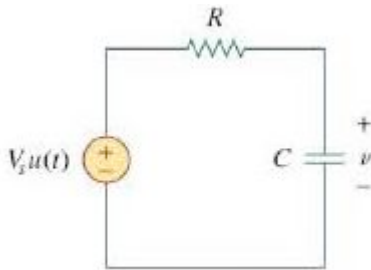
If the circuit has a voltage source then

Apply this variation  $V(t) = V_s + (V_0 - V_s)e^{-t/\tau}$

where  $V_s$  = the source voltage and

$V_0$  is the initial charge on the capacitor

The capacitor will reach maximum charge in about 5 $\tau$



**Inductance RL circuits**

1. Find a bit different to capacitance in that division is used

$$\tau = L/R$$

2.  $i(t) = i_s + (i_0 - i_s)e^{-t/\tau}$  As with capacitance

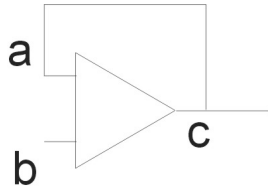
$K = i_0$  = the initial Current

$i_s$  = the Source Current

$\tau$  = Tau is the time constant

t = is the point in time

NB: To see how  $V = Ke^{-t/\tau}$  has come about see Helen Middletons MAS182 tutorial on page MT26.2



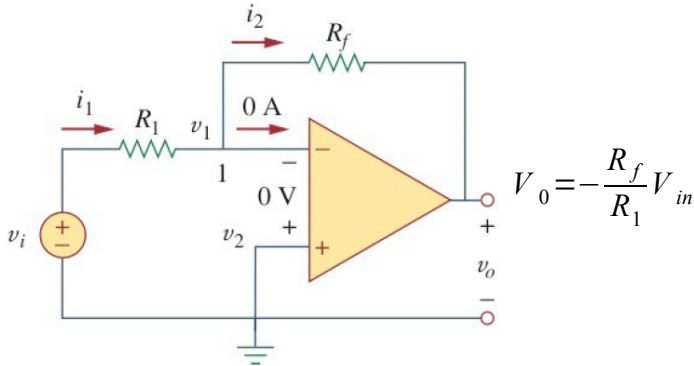
1. The voltage at 'c' is varied so that 'a' and 'b' remain equal.
2. If b is connected to ground (0v) then 'c' will draw in or push out current so that 'a' and 'b' stay the same.
3. 'b' does not have to be connected to ground though. It could be connected to a power source like in a difference amplifier.

When calculating KCL at 'a' assume that 'a' has the same voltage as 'b'.

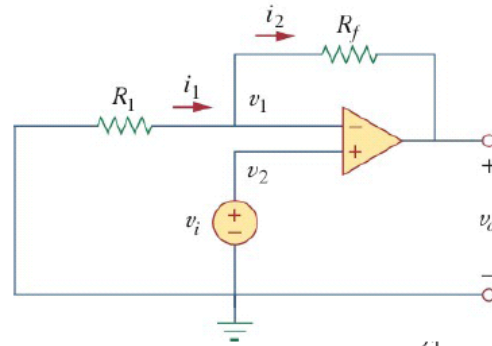
**To calculate what the amplification amount is**

1.  $V_{\text{difference of 'a' and 'b'}} = V_d = V_b - V_a$
2. A = Amplification
3. A in an ideal op amp is infinity. In a real op amp it is about  $10^5$  to  $10^{13}$
4.  $V_c = AV_d = A(V_b - V_a)$

**Inverting Amplifier**

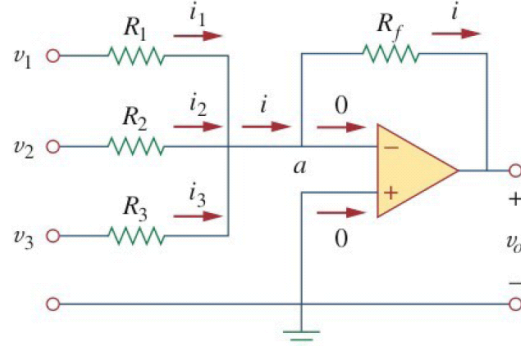


**Non - Inverting Amplifier**



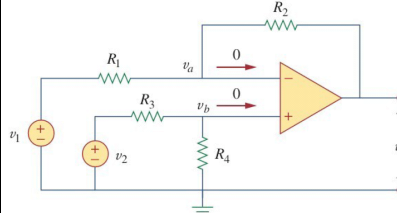
$$V_o = \left(1 + \frac{R_f}{R_1}\right) V_{in}$$

**Summing Amplifier**



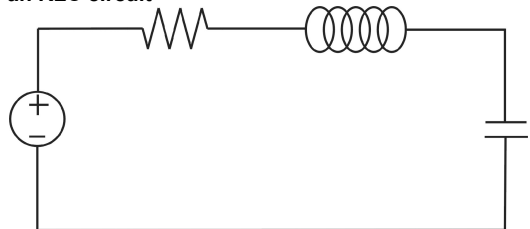
$$V_o = -\left(\frac{R_f}{R_1} V_1 + \frac{R_f}{R_2} V_2 + \frac{R_f}{R_3} V_3 + \right)$$

**Difference Amplifier**



$$V_o = \frac{R_2 \left(1 + \frac{R_1}{R_2}\right)}{R_1 \left(1 + \frac{R_3}{R_4}\right)} V_2 - \frac{R_2}{R_1} V_1$$

**Second Order Circuits To Solve an RLC circuit**



**The Short Versi'on**

- 1 Apply KVL to the RLC circuit.
2. Differentiate
3. Mash it around till you get a quadratic.
3. Insert those values into the cookbook 2nd order equation.
5. Bobsyerunki.

**The Longer Version**

1. Apply KVL
2.  $V_s = V_R + V_L + V_C$

3. Express each component as a voltage

$$V_s = Ri + L \frac{di}{dt} + \frac{1}{C} \int_{t_0}^t V_c dt + v(t_0)$$

4. Now we need to get rid of the integrated section.

5. Do this by differentiating the whole equation with respect to 't'.

$$\frac{dV_s}{dt} = R \frac{di}{dt} + L \frac{d^2i}{dt^2} + \frac{V_c}{C}$$

6. After rearranging and substituting in  $i = C \frac{dv}{dt}$  we get

$$\frac{d^2v}{dt^2} + \frac{R}{L} \frac{dv}{dt} + \frac{v}{LC} = \frac{V_s}{LC}$$

7. We know that the answer for  $v(t) = Ae^{st} + B$  where  $B = \frac{V_s}{LC}$ .

The B section of this is a little confusing. It represents the source forcing voltage in an RLC circuit. In a source free RLC circuit the B = 0, so is not shown.

7.1. So we need to substitute

$$v(t) = Ae^{st} \text{ and}$$

$$\frac{dv}{dt} = Ase^{st} \text{ and}$$

$$\frac{d^2v}{dt^2} = As^2e^{st} \text{ into the equation at 6.}$$

7.2. This gives  $As^2e^{st} + \frac{R}{L}Ase^{st} + \frac{Ae^{st}}{LC} = \frac{V_s}{LC}$

7.3. From here we can factor out the  $Ae^{st}$  giving

$$Ae^{st} \left( s^2 + \frac{R}{L}s + \frac{1}{LC} \right) = \frac{V_s}{LC}$$

Note  $Ae^{st}$  can never really be zero so we must find where the quadratic term inside the brackets equals zero.

7.4. So now the section in the brackets is in the general form of a quadratic.

$$\left( s^2 + \frac{R}{L}s + \frac{1}{LC} \right) = 0 \text{ ie } as^2 + bs + c = 0$$

Now we can apply the quadratic formula to this.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ This will give us 2 answers. These 2 answers}$$

are the  $A_1$  and  $A_2$  in the following equation at steps 9, 10 & 11.. But first

7.5.  $\omega_0 = \frac{1}{\sqrt{LC}} \quad \alpha = \frac{R}{2L}$

Omega is measured in radians/second which is the angular Frequency

8. Figure out the amount of damping =  $\zeta = Zeta = \frac{\alpha}{\omega}$  if

$$\zeta = 1 \rightarrow \text{Critically Damped}$$

$$\zeta < 1 \rightarrow \text{Under Damped}$$

$$\zeta > 1 \rightarrow \text{Over Damped}$$

8.1. For the B in the following equations use  $B = \frac{V_s}{LC}$

9. For Over damped use  $v_i(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} + B$

10. For Critically Damped use  $v_i(t) = (A_1 + A_2 t) e^{-at} + B$

11. Under Damped

$$v_i(t) = (A_1 \cos \omega_d t + A_2 \sin \omega_d t) e^{-at} + B$$

12. Note that  $s = \frac{-t}{\tau}$  which converts  $e^{st}$  to  $e^{\frac{-t}{\tau}}$

2<sup>nd</sup> order circuits – The refined version – Use the method outlined on page 178 in Hambley.

$$1. \quad \omega_0 = \frac{1}{\sqrt{LC}} \quad \alpha = \frac{R}{2L}$$

Omega is measured in radians/second which is the angular Frequency

2. Figure out the amount of damping =  $\zeta = Zeta = \frac{\alpha}{\omega}$  if

$$\zeta = 1 \rightarrow \text{Critically Damped}$$

$$\zeta < 1 \rightarrow \text{Under Damped}$$

$$\zeta > 1 \rightarrow \text{Over Damped}$$

3. Find  $S_1$  and  $S_2$ . If alpha and Omega are the same then  $s_1$  and  $s_2$  will be the same. ( use the  $s_1$  equation below)

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2} \quad s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

4.

**To find Tau in RLC circuits.**

See page 178 in Hambley and for each case:

1. OVERDAMPED (eqn 4.74) depends on which ever of  $s_1$  and  $s_2$  is smallest

$$\Rightarrow \tau = -1 / \min(s_1, s_2)$$

2. CRITICALLY DAMPED (eqn 4.75) only depends on  $s_1$  (since root is repeated)

$$\Rightarrow \tau = -1 / s_1$$

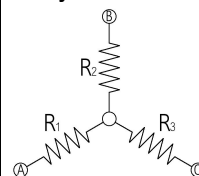
3. UNDERDAMPED (eqn 4.76) depends on alpha since this is the damping envelope

$$\Rightarrow \tau = 1 / \alpha$$

As with first order the the function will approach the final value after approximately  $5 * \tau$ .

**Parallel RLC circuits** see page 186

**Delta y transforms**



Delta to Wye

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

Wye to Delta

$$R_A = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_1}$$

$$R_B = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_2}$$

$$R_C = \frac{R_1 R_2 + R_1 R_3 + R_2 R_3}{R_3}$$

**Super mesh**

**Super Node**

If asked for a differential equation always apply a KVL or a KCL.

Siemens = Inverse Ohms

After one time constant the voltage should be 36% of the original

**Nodal Analysis**

With the currents going in and out of a node.

Do a positive or negative for the source arm.

Then just do a positive for all the other arms without sources.

Be consistent with assigning negative or positive to source arms

See example from exam 2002 Q2.

Then arrange your simultaneous equations for each node.

Do your matrix analysis.

**Phasors** : -A representation of the current and the voltage as a vector.(Complex Number)

-Allows finding of steady state

**Currents sometimes can be given with a Hz (Hertz) value** - eg (10volts at 100Hz)  
To convert this to the general form below you may need to apply this

$$\text{Frequency} = f = \frac{1}{T} \text{ measured in Hertz (Hz)}$$

$$\text{Angular Frequency} = \omega = 2\pi f \text{ measured in radians per second.}$$

**General form of a Sinusoidal Current**

$$v(t) = v_m \cos(\omega t + \theta_1)$$

The Phasor Representation of this current is

$V_1 = V_m \angle \theta_1$  (polar) (eng243\_lect02.pdf page 4 or Hambley pg 207)

- $V_m$  = Peak Value (amplitude)
- $\omega$  = Angular Frequency (in Radians per second)
- $\theta$  = Phase Angle (In Degrees)
- T = Period
- f = frequency

Frequency =  $f = \frac{1}{T}$  measured in Hertz (Hz)

Or  $f = \frac{\omega}{2\pi}$

Angular Frequency =  $\omega = 2\pi f$  measured in radians per second.

Functions are related by the identity (pg201)

$\sin(z) = \cos(z - 90^\circ)$

**Adding Sinusoids (which have the same frequency)**

- Convert the current to a Phasor in Polar complex number format
- Add the Phasors (complex numbers)
- Then convert back to the general form by

$v(t) = \sqrt{V_m^2 + \theta_1^2}$

**Phasor Representation can be in 3 forms**

- $V_1 = V_m \angle \theta$  (polar)
- $V_1 = x + yj$  (rectangular)
- $V_1 = r \cdot e^{j\theta}$  (exponential)

To convert between forms see (eng243\_lect02.pdf page 4)

**CIVIL**

- Capacitor
- Current leads voltage
- Inductor
- Voltage leads current

**Impedance and Resistance**

- V=IR in Ohms law
- In AC
- Z = R
- V=IZ

To find  $\theta$  in degrees  $\sin^{-1}\left(\frac{O}{H}\right) = \theta$

To convert degrees to Radians  $(x^\circ / 180) \pi = \text{Radians}$

eg  $(15^\circ / 180) \pi = 0.2617 \text{ Radians}$

To convert Radians to Degrees  $x^\circ = \frac{\text{Radians}}{\pi} (180^\circ)$

**Complex impedance of Inductors**

$z_L = \frac{V_L}{I_L} = j\omega L$

**Reactance of Inductors**

$X_L = \omega L$

Inductance as an impedance phasor:  $z_L = j\omega L = \omega L < 90^\circ$

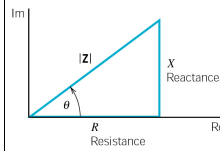
**Capacitance**

$z_C = \frac{V_C}{I_C} = \frac{1}{j\omega C}$

Capacitance as an impedance phasor:  $z_C = \frac{1}{j\omega C} = \frac{1}{\omega C} < -90^\circ$

**Reactance :**

$X_C = \frac{1}{\omega C}$



$(Z=R+jX)$ .

R ~ DC resistance, X ~ AC reactance.

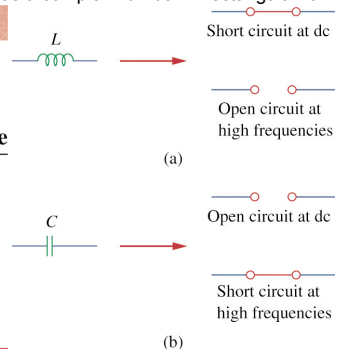
Impedance is a phasor, often represented as a complex number in rectangular form

TABLE 9.3

**Impedances and admittances of passive elements.**

Element	Impedance	Admittance
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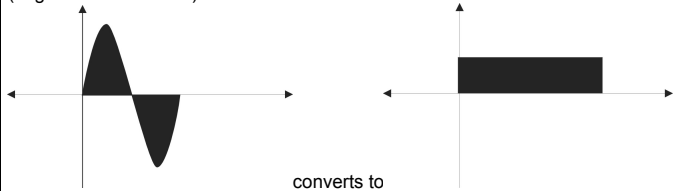
R	$Z = R$	$Y = \frac{1}{R}$
L	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
C	$Z = \frac{1}{j\omega C}$	$Y = j\omega C$



**RMS -Why do we do rms to AC circuits??**

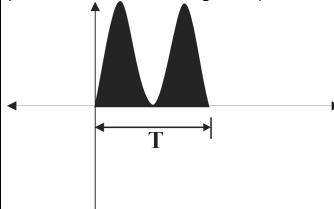
The reason we do RMS is to give the voltage or current of an AC circuit as the DC equivalent.

So there fore what we are trying to do is get the area in one sin wave revolution (negative and Positive) and convert it to a flat line with the same area under it.



**RMS = Root Means Squared.**

1. Square first (This pushes the sine wave into the positive axes so that we have all positive areas to add together.)



2. Then Mean it (or find the value under this curve by integration)
3. Then convert this area to a square block. Because we know the width (width = T)

and area of a block is width x height. Or  $a = w \times h$  rearrange to  $\frac{a}{w} = h$ .

The w = T and the a = 1 x (what ever the area under the curve was).

So  $\frac{1}{T} = h$

4. Then because we squared at the beginning we have to Square root at the end to undo this function.

**Voltage rms (Root-mean-square)** pg 202

$V_{rms} = \sqrt{\frac{1}{T} \int_0^T v^2(t) dt}$

**Current rms (Root-mean-square)** pg 202

$I_{rms} = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt}$  or the simplified version is  $I_{rms} = \frac{I_m}{\sqrt{2}}$

**RMS Value of a Sinusoid** pg 203

$V_m$  = the peak voltage

$V_{rms} = \frac{V_m}{\sqrt{2}}$  or for current use  $I_{rms} = \frac{I_m}{\sqrt{2}}$

**Voltage rms** - for triangular waveforms

$V_{rms} = \frac{V_m}{\sqrt{3}}$  and for square it's just  $V_m$

Eulers rule to Remember

$$e^{j\theta} = \cos \theta + j \sin \theta$$

### Eulers identity

$$e^{j\pi} + 1 = 0$$

When given 2 AC currents in the form of

$$v(t) = 10 \cos(\omega t + 0) + 10 \cos(\omega t - 90)$$

and they need to be added convert them first to polar form

$$v(t) = 10 \angle 0^\circ + 10 \angle -90^\circ$$

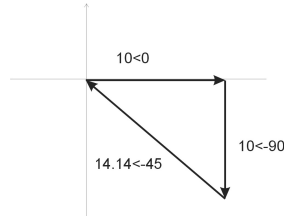
- Draw diagrams at this point to visualise the equation.

- Add the real parts separately to the imaginary parts.

$$v(t) = 14.14 \angle -45^\circ$$

- Convert this back into rectangular form

$$v(t) = 14.14 \cos(\omega t - 45^\circ)$$



To multiply complex numbers in polar form,

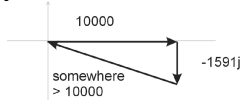
Multiply the r parts  
Add the angle parts

To divide complex numbers in polar form,

Divide the r parts  
Subtract the angle parts

To convert a rectangular format to a polar

Say  $Z_t = 10000 - 1591j$



- First draw a diagram

- Then use Pythagoras theorem to figure out the real part

$$v(t) = \sqrt{10000^2 + (-1591j)^2}$$

$$\hat{i} 10125 \angle ?$$

- Then use SOHCAHTOA to find theta

$$-\tan \theta = \frac{-1591.55}{10000} \text{ which is } \theta = -9^\circ$$

$$\therefore v(t) = 10125 \angle -9^\circ$$

### Multiplication of Complex Numbers

1.  $5(2 + 7j) = 10 + 35j$
2.  $(6 - j)(5j) = 30j - 5j^2 = 5 + 30j$
3.  $(2 - j)(3 + j) = 6 - 3j + 2j - j^2 = 6 - (-1) - j = 7 - j$

### Division of Complex Numbers (intmath.com)

- Multiply out the denominator by its conjugate

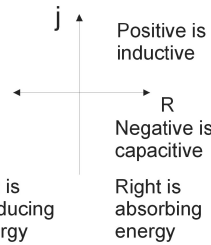
### Phasers

When multiplying phasers  
Multiply real and add angles  
When dividing phasers  
Divide real and subtract angles

$$\begin{aligned} \frac{1 - \sqrt{-4}}{2 + 9j} &= \frac{1 - 2j}{2 + 9j} \times \frac{2 - 9j}{2 - 9j} \\ &= \frac{2 - 9j - 4j + 18j^2}{4 - 81j^2} \\ &= \frac{-16 - 13j}{4 + 81} \\ &= \frac{-16 - 13j}{85} \end{aligned}$$

### Identity for j

$$\frac{A}{j} \cdot \frac{j}{j} = -jA \quad \text{or} \quad \frac{1}{j} = -j \quad \text{or} \quad \frac{1}{-j} = j$$



### When doing Nodal Analysis

Always write the KCL basic equation as equal to 0 first.

$$\text{ie. } 0 = I_1 + I_2 - I_3$$

Then rearrange it to all positives if need be

ie.  $I_3 = I_1 + I_2$  but doing this can cause you to muddle signs up. There fore keep everything as equal to 0.

Then substitute ohms law into  $0 = I_1 + I_2 - I_3$  To make

$$0 = \frac{V}{R} + \frac{V}{R} - \frac{V}{R}$$

### Big thing to remember. If you keep it all equal to 0

- then you can look at the direction of the arrow of the current.

- The arrow points in the direction of the smaller voltage.

- There fore you minus the bigger voltage from the smaller one.

$$0 = \frac{V_2 - V_1}{R} + \frac{V_1 - 0}{R} - \frac{V_1 - V_3}{R} \quad \text{Now from here you can}$$

rarrange to isolate the V's

Why do we convert the sin wave to a cos wave?

### MATLAB

x = REAL(z)  
y = REAL(z)  
|z| = ABS(z)  
<z = ANGLE(z)\*180/pi

### Maximum Power Transfer – Using Thevenin rule (page 239 Hamb)

The maximum impedance used to give the maximum load in an AC system is the complex conjugate of the Thevenin resistance.

### Thevenin and Norton Equivalent circuits – pg 84

1. The Thevenin Source volatage is equal to the open circuit voltage.
2. a. The Thevenin resistance is equal to the open circuit voltage divided by the short circuit current.

$$R_{TH} = \frac{V_{open-circuit}}{I_{short-circuit}} \quad \text{Or}$$

b. with INDEPENDENT (note : but not dependant) sources in the circuit you can

just zero the sources and take the open circuit resistance.

- (i) Zero all sources  
Zero current = open circuit  
Zero voltage = short circuit
- (ii) Find  $R_{TH}$  for this @ ab
- (iii) Put sources back
- (iv) Short circuit ab
- (v) Find current across short using nodal or any other means =  $I_{TH}$
- (vi) Draw thevenin equivalent circuit using formula at a.

c..Pg – 89 For circuits with DEPENDANT sources method b. can not be done.

### To convert polar ↔ rectangular formats on calc. Pg 46 Kemp

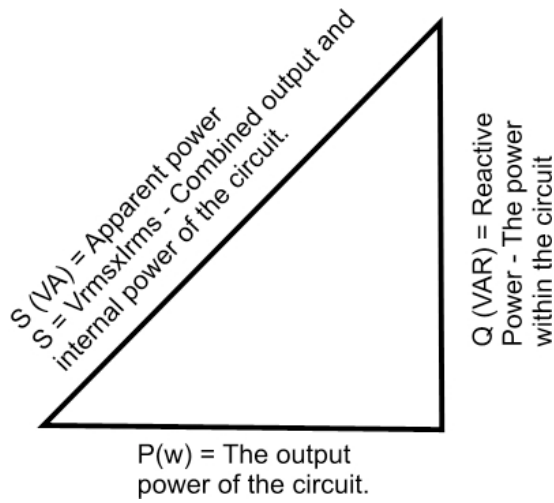
Shift → SETUP → select angle type required

OPTN → F6 → F5 (ANGL) → F6 →

To convert to polar choose POL and enter rectangular coords separated by a comma.

To convert to rectangular choose REC and enter polar coords.

### Power Factor Triangle - Hambley pg228 - 230



Q = The power within the circuit generally held by inductance and Capacitance. Q is called reactive power. It has the unit of var, rather than watt. It is positive for inductive elements, when current lags voltage ( $\theta_I < \theta_V$ ); and negative for capacitive elements when current leads voltage.  
 P = The real or **active** power output by the circuit without Q.  
 S = Is the apparent power which is a combination of Q and P using trig calculations.

To find the current from the power triangle : p235 Hambley

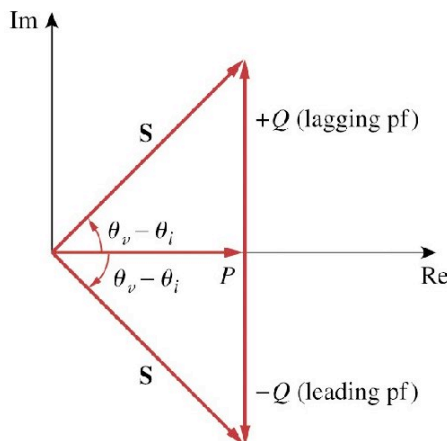
$$S = V_{rms} I_{rms}$$

$$I_{rms} = \frac{V_{rms} I_{rms}}{V_{rms}}$$

Finding the impedance or reactance from the power triangle : pg236 Hamb Eg – 5.9

$$Z_{eq} = X_{eq} = \frac{V_{rms}^2}{Q_{\text{From Power Triangle}}}$$

Power Factor Triangle Lect 2010-04-Power\_Analysis.pdf - pg28



$$S = \frac{1}{2} VI^* \quad \text{Page 23 - r2010-04-Power_Analysis.pdf}$$

**Complex Conjugate**

$$I^* = \text{The complex conjugate of } I$$

This changes the sign of the complex part.

**Power Factor** hamb pg 228

$$PF = \cos(\theta) \quad (5.61)$$

where  $\theta = \theta_v - \theta_i$

Stated as a percentage (ie power factor of 0.9 lagging or leading)

**Note from Hambley pg 228 – paragraph after equation (5.62)**

A leading or lagging load will be stated when the power factor is described.

Leading current (Capacitive Load) ( $\theta$  negative)

Lagging Current (Inductive Load) ( $\theta$  positive)

**WYE circuits**

Line to line voltages – pg 248 Hamb

$$V_L = \sqrt{3} V_Y \quad (5.96)$$

$$V_{ab} = V_{an} \times \sqrt{3} < 30^\circ \quad (5.97)$$

The line to line voltage is  $\sqrt{3}$  x the line to neutral voltage.

This is only for positive sequence phasors. For negative sequence use

$$V_{ab} = V_{an} \times \sqrt{3} < -30^\circ$$

Add diagram here from WSAC-19 or page 249. hambley.

**Wye to Delta load conversion** hamb pg 251

$$Z_\Delta = 3 Z_Y \quad (5.100)$$

$$V_L = \sqrt{3} V_Y \quad (5.96) \text{ pg 248}$$

**Thompsons Formula** From Lab5 week 5

This is used to find the resonant frequency of an RLC circuit.

Applies to series and parallel circuits – p16 2010-07-Filters\_Resonance.pdf

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ (Hz)}$$

**Sinusoidal Currents and Voltages** – p14 – 2010-02-

Sinusoids\_Phасors\_Impedances.pdf

$$\sin(x) = \cos(x - 90^\circ)$$

**Eulers Identity** – pg 17 2010-02-Sinusoids\_Phасors\_Impedances.pdf

$$\cos(\omega t + \theta) = e^{j(\omega t + \theta)} = \cos \theta + j \sin \theta$$

**Phase Sequence** – pg 244 Hambley

abc is positive  
 acb is negative

**Transfer Functions** – pg 273 Hamb

$$H(f) = \frac{V_{out}}{V_{in}} \text{ where } f \text{ is the Frequency and } V_{out} / V_{in} \text{ are phasors}$$

**Transfer Functions decibels** – pg 284 Hamb

**Logarithmic Frequency Scales** – pg 287

Octave 2:1 ratio

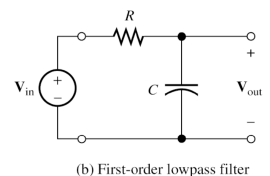
Decade 10:1 ratio

**First order low pass filters** pg 279 Hamb

Worked example – pg 281 Hamb

$$f = \frac{\omega}{2\pi}$$

$$f_B = \frac{1}{2\pi RC} \text{ if the capacitor}$$



(b) First-order lowpass filter

is swapped out for an inductor the equation becomes

$$f_B = \frac{R}{2\pi L} \text{ - p296 Hamb}$$

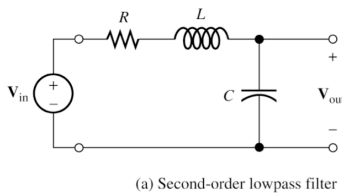
$$H(f) = \frac{1}{1 + j(f/f_B)}$$

Once you have  $H(f)$  substitute it into the equation

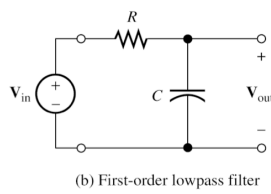
$$H(f) = \frac{V_{out}}{V_{in}} \rightarrow V_{out} = H(f) \times V_{in} \text{ to get your } V_{out}$$

Magnitude and Phase plots of low pass – p280 hamb

Bode Plots of low pass – p290



(a) Second-order lowpass filter



(b) First-order lowpass filter

**First order high pass filters** pg 292 Hamb

$$H(f) = \frac{j(f/f_B)}{1 + j(f/f_B)} \quad \text{p292 Hamb}$$

$$f_B = \frac{1}{2\pi RC}$$

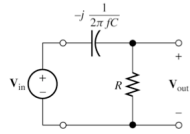


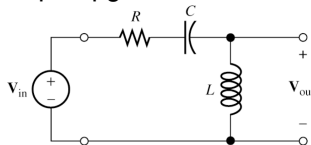
Figure 6.19 First-order highpass filter.

Magnitude and Phase plots of high pass – p293 hamb  
Bode Plots of high pass - p295

p281 Hamb

$f_B$  is also known as the **half power frequency** or **Corner Frequency** or **Break Frequency**.  
Which is  $1/\sqrt{2} \approx 0.707$  the maximum value.

**Second order High Pass Filter** 2010-07-Filters\_Resonance.pdf -pg28



**Decibels in relation to Hf** pg 284 Hamb

The expression  $H(f) = \frac{V_{out}}{V_{in}}$  is just a ratio, so when we make a bode plot of it we need to convert it into decibels on a logarithmic scale. The formula for this is :

$|H(f)_{dB}| = 20 \log|H(f)|$  in Matlab it would look like :

$$Hdb = 20 * \log_{10}(\text{abs}(Hf));$$

An example Matlab file is on p299 Hamb.

**Voltage and current ratios** pg 748 hamb

$$v_2(t) = \frac{N_2}{N_1} v_1(t) \quad (15.50)$$

$$I_{2rms}(t) = \frac{N_2}{N_1} I_{1rms}(t) \quad (15.56)$$

**Impedance Transformations** pg 752

$$Z'_L = \left(\frac{N_1}{N_2}\right)^2 Z_L \quad (15.62)$$

**Developed power** – p793 Hamb

Is the power delivered to the armature and converted to mechanical power.

$$P_{dev} = I_A E_A = \omega_m T_{dev}$$

**Q Factor** – 2010-07-Filters\_Resonance.pdf -pg17

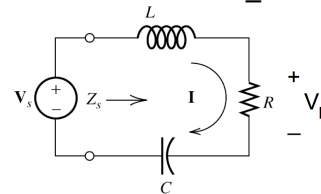
**Get Gareth to explain this further**

The quality factor  $Q_s$  of a series circuit is defined to be the ratio of the resistance to the reactance of the inductance at the resonant frequency.

$$f_0 = \frac{1}{2\pi\sqrt{LC}} \text{ (Hz)} \quad (6.40)$$

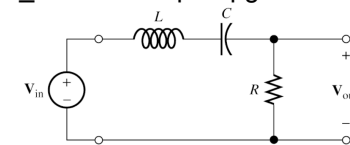
$$Q_p = \frac{R}{2\pi f_0 L} \quad (6.41)$$

**Band Pass Filter** 2010-07-Filters\_Resonance.pdf -pg19



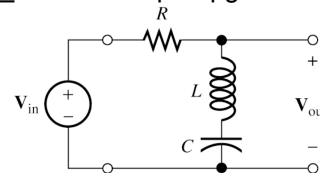
**Second Order Band Pass Filter**

2010-07-Filters\_Resonance.pdf -pg29



**Second Order Band-Reject (Notch) Filter**

2010-07-Filters\_Resonance.pdf -pg30



**Parallel Resonance** – p306 Hamb