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(a)

$$\sqrt{xy} = 1 + x^2y$$

implicit differentiation

$$xy^{\frac{1}{2}} = 1 + x^2y$$

$$\left(\frac{d}{dx} x^{\frac{1}{2}}\right) y^{\frac{1}{2}} + x^{\frac{1}{2}} \left(\frac{d}{dy} y^{\frac{1}{2}} \cdot \frac{dy}{dx}\right) = \left(\frac{d}{dx} x^2\right) y + x^2 \left(\frac{d}{dy} y \cdot \frac{dy}{dx}\right)$$

$$\frac{x^{-\frac{1}{2}}}{2} \cdot y^{\frac{1}{2}} + x^{\frac{1}{2}} \cdot \frac{y^{-\frac{1}{2}}}{2} \cdot \frac{dy}{dx} = 2xy + x^2 \frac{dy}{dx}$$

$$\frac{x^{-\frac{1}{2}} y^{\frac{1}{2}}}{2} + \frac{x^{\frac{1}{2}} y^{-\frac{1}{2}}}{2} \cdot \frac{dy}{dx} = 2xy + x^2 \frac{dy}{dx}$$

$$\frac{x^{-\frac{1}{2}} y^{\frac{1}{2}}}{2} + \frac{x^{\frac{1}{2}} y^{-\frac{1}{2}}}{2} \cdot \frac{dy}{dx} - 2xy - x^2 \frac{dy}{dx} = 0$$

$$\frac{x^{-\frac{1}{2}} y^{\frac{1}{2}}}{2} + 2xy + \frac{dy}{dx} \left[\frac{x^{\frac{1}{2}} y^{-\frac{1}{2}}}{2} - x^2 \right] = 0$$

$$\frac{dy}{dx} = \frac{-\frac{x^{-\frac{1}{2}} y^{\frac{1}{2}}}{2} - 2xy}{\frac{x^{\frac{1}{2}} y^{-\frac{1}{2}}}{2} - x^2}$$